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A STUDY OF PHASE MEASURING
CIRCUITS AND TECHNIQUES

PHILIP ROBERT LAURIAT

1953

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A STUDY OF
PHASE MEASURING CIRCUITS AND TECHNIQUES

By

Philip Robert Lauriat
Lieutenant, United States Navy

Submitted in partial fulfillment
of the requirements
for the degree of

MASTER OF SCIENCE

IN

ENGINEERING ELECTRONICS

United States Naval Postgraduate School
Monterey, California
1953

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This work is accepted as fulfilling
the thesis requirements for the degree of

MASTER OF SCIENCE

IN

ENGINEERING ELECTRONICS

for the
United States Postgraduate School

TABLE OF CONTENTS

	Page
Chapter I Introduction	1
Chapter II Visual Methods of Phase Measurement	5
Chapter III Considerations of an Electronic Phasemeter	19
Chapter IV Standards for Phase Measurement and Calibration	45
Chapter V Conclusions	53
Bibliography	56
Appendix I Precision Measurement of Phase Coincidence Using a Cathode Ray Oscilloscope	58
Appendix II Phase Invariance During the Heterodyne Process	59
Appendix III Analysis of the Cathode Coupled Clipper Circuit	61
Appendix IV Constant Attenuation Phase Shift Networks	66
Appendix V Equivalent Circuit for a Capacitance Goniometer	67

Page	Chapter
1	Chapter I: Introduction
2	Chapter II: General Principles of the Theory
10	Chapter III: The Structure of the Matter
15	Chapter IV: The Properties of the Matter
20	Chapter V: The Properties of the Matter
25	Chapter VI: The Properties of the Matter
30	Chapter VII: The Properties of the Matter
35	Chapter VIII: The Properties of the Matter
40	Chapter IX: The Properties of the Matter
45	Chapter X: The Properties of the Matter
50	Chapter XI: The Properties of the Matter
55	Chapter XII: The Properties of the Matter
60	Chapter XIII: The Properties of the Matter
65	Chapter XIV: The Properties of the Matter
70	Chapter XV: The Properties of the Matter

LIST OF ILLUSTRATIONS

Figure		Page
1.	Lissajoux Phase Pattern	4
2.	Phase Error in Visual Measurement due to Trace Thickness	6
3.	Phase Error in Visual Measurement due to Harmonic Distortion	7
4.	Observational Error due to Trace Thickness	4
5.	Effect of Non-linear Sweep on the Intercept Method of Phase Measurement	9
6.	Visual Phase Measurement by Circular Trace	13
7.	A Laboratory Application of Z-Modulation Phase Measurement	15
8.	An R-C Lattice Phase Shift Network	18
9.	Output Phase Characteristic for a Pair of Networks Used to Provide a Constant Phase Difference over a Wide Band of Frequencies	18
10.	A Basic L-C-R Constant Attenuation Phase Shift Network	21
11.	General Characteristic of a Constant Phase Difference Network	21
12.	The General Feedback Amplifier	25
13.	An R-C Feedback Amplifier Precision Phasemeter	25
14.	A Pentode Wave Squaring Circuit	28
15.	Apparent Phase Error due to Dissymmetry in Wave Squaring	28
16.	The Cathode Coupled Clipper Circuit	30
17.	Performance Characteristics of the Cathode Coupled Clipper	31
18.	Phase Discrimination by Addition of Voltages	33
19.	Output of a Peak Detecting Discriminator	34
20.	The Diode Bridge or Ring Modulator	36

THE HISTORY OF THE

Page	Chapter	Page
1	THE HISTORY OF THE	1
2	THE HISTORY OF THE	2
3	THE HISTORY OF THE	3
4	THE HISTORY OF THE	4
5	THE HISTORY OF THE	5
6	THE HISTORY OF THE	6
7	THE HISTORY OF THE	7
8	THE HISTORY OF THE	8
9	THE HISTORY OF THE	9
10	THE HISTORY OF THE	10
11	THE HISTORY OF THE	11
12	THE HISTORY OF THE	12
13	THE HISTORY OF THE	13
14	THE HISTORY OF THE	14
15	THE HISTORY OF THE	15
16	THE HISTORY OF THE	16
17	THE HISTORY OF THE	17
18	THE HISTORY OF THE	18
19	THE HISTORY OF THE	19
20	THE HISTORY OF THE	20
21	THE HISTORY OF THE	21
22	THE HISTORY OF THE	22
23	THE HISTORY OF THE	23
24	THE HISTORY OF THE	24
25	THE HISTORY OF THE	25
26	THE HISTORY OF THE	26
27	THE HISTORY OF THE	27
28	THE HISTORY OF THE	28
29	THE HISTORY OF THE	29
30	THE HISTORY OF THE	30
31	THE HISTORY OF THE	31
32	THE HISTORY OF THE	32
33	THE HISTORY OF THE	33
34	THE HISTORY OF THE	34
35	THE HISTORY OF THE	35
36	THE HISTORY OF THE	36
37	THE HISTORY OF THE	37
38	THE HISTORY OF THE	38
39	THE HISTORY OF THE	39
40	THE HISTORY OF THE	40

Figure		Page
21.	Technology Institute Phasemeter Principle Illustrated by Time Voltage Waveforms (On-time Off-time Phasemeter)	39
22.	Typical Summing Amplifiers	41
23.	A Phase Sensitive Circuit Using a Dual Control Tube	42
24.	Method of Obtaining the Base Frequency for a Standard Lag Line for Use as a Phase Standard and Measuring Device	42
25.	Calibration Curve for a 20 Kilocycle Standard Lag Line	46
26.	Resistance Goniometer and Typical Quadrature Voltage Generator Circuit	48
27.	Resistance Goniometer Output Phase Error	49
28.	Capacitance Goniometer and Equivalent Circuit	51
29.	A Proposed Precision Phasemeter	55
30.	Reduction of Several Phase Shift Networks to Basic Form	66

Page	Subject	Page
11	Technology Institute, University of Illinois	11
12	United Nations Institute	12
13	United Nations Institute	13
14	United Nations Institute	14
15	United Nations Institute	15
16	United Nations Institute	16
17	United Nations Institute	17
18	United Nations Institute	18
19	United Nations Institute	19
20	United Nations Institute	20
21	United Nations Institute	21
22	United Nations Institute	22
23	United Nations Institute	23
24	United Nations Institute	24
25	United Nations Institute	25
26	United Nations Institute	26
27	United Nations Institute	27
28	United Nations Institute	28
29	United Nations Institute	29
30	United Nations Institute	30
31	United Nations Institute	31
32	United Nations Institute	32
33	United Nations Institute	33
34	United Nations Institute	34
35	United Nations Institute	35
36	United Nations Institute	36
37	United Nations Institute	37
38	United Nations Institute	38
39	United Nations Institute	39
40	United Nations Institute	40
41	United Nations Institute	41
42	United Nations Institute	42
43	United Nations Institute	43
44	United Nations Institute	44
45	United Nations Institute	45
46	United Nations Institute	46
47	United Nations Institute	47
48	United Nations Institute	48
49	United Nations Institute	49
50	United Nations Institute	50
51	United Nations Institute	51
52	United Nations Institute	52
53	United Nations Institute	53
54	United Nations Institute	54
55	United Nations Institute	55
56	United Nations Institute	56
57	United Nations Institute	57
58	United Nations Institute	58
59	United Nations Institute	59
60	United Nations Institute	60
61	United Nations Institute	61
62	United Nations Institute	62
63	United Nations Institute	63
64	United Nations Institute	64
65	United Nations Institute	65
66	United Nations Institute	66
67	United Nations Institute	67
68	United Nations Institute	68
69	United Nations Institute	69
70	United Nations Institute	70
71	United Nations Institute	71
72	United Nations Institute	72
73	United Nations Institute	73
74	United Nations Institute	74
75	United Nations Institute	75
76	United Nations Institute	76
77	United Nations Institute	77
78	United Nations Institute	78
79	United Nations Institute	79
80	United Nations Institute	80
81	United Nations Institute	81
82	United Nations Institute	82
83	United Nations Institute	83
84	United Nations Institute	84
85	United Nations Institute	85
86	United Nations Institute	86
87	United Nations Institute	87
88	United Nations Institute	88
89	United Nations Institute	89
90	United Nations Institute	90
91	United Nations Institute	91
92	United Nations Institute	92
93	United Nations Institute	93
94	United Nations Institute	94
95	United Nations Institute	95
96	United Nations Institute	96
97	United Nations Institute	97
98	United Nations Institute	98
99	United Nations Institute	99
100	United Nations Institute	100

TABLE OF SYMBOLS AND ABBREVIATIONS

a, b, c, k	- Coordinates, points of intersection, constants
$A, B, C,$	- Coordinates, points of intersection, amplitudes, constants
ω	- Electrical angular frequency
ϕ	- Electrical phase angle
ψ	- Angular difference between electrical phase angles
ϵ	- An incremental error
$s, g, h, p,$	- Design parameters
K	- Gain or Amplitude factor
θ	- Absolute mechanical or electrical angle
τ	- Time increment
T	- Period, time constant
V, E	- Peak voltage amplitude
v, e	- Instantaneous voltage as function of position, time or both
Z	- General expression for an impedance
μ	- Incremental amplification factor, design parameter
l	- Length, incremental inductance
f_0	- Geometrical mean frequency in a frequency band
$f_{1,2}, \text{ etc.}$	- Resonant frequency

TABLE OF CONTENTS

1. Introduction	1
2. Objectives	2
3. Scope	3
4. Methodology	4
5. Results	5
6. Discussion	6
7. Conclusion	7
8. References	8
9. Appendix	9
10. Glossary	10
11. Bibliography	11
12. Index	12
13. Acknowledgements	13
14. Executive Summary	14
15. Abstract	15
16. Introduction	16
17. Objectives	17
18. Scope	18
19. Methodology	19
20. Results	20
21. Discussion	21
22. Conclusion	22
23. References	23
24. Appendix	24
25. Glossary	25
26. Bibliography	26
27. Index	27
28. Acknowledgements	28
29. Executive Summary	29
30. Abstract	30
31. Introduction	31
32. Objectives	32
33. Scope	33
34. Methodology	34
35. Results	35
36. Discussion	36
37. Conclusion	37
38. References	38
39. Appendix	39
40. Glossary	40
41. Bibliography	41
42. Index	42
43. Acknowledgements	43
44. Executive Summary	44
45. Abstract	45
46. Introduction	46
47. Objectives	47
48. Scope	48
49. Methodology	49
50. Results	50
51. Discussion	51
52. Conclusion	52
53. References	53
54. Appendix	54
55. Glossary	55
56. Bibliography	56
57. Index	57
58. Acknowledgements	58
59. Executive Summary	59
60. Abstract	60
61. Introduction	61
62. Objectives	62
63. Scope	63
64. Methodology	64
65. Results	65
66. Discussion	66
67. Conclusion	67
68. References	68
69. Appendix	69
70. Glossary	70
71. Bibliography	71
72. Index	72
73. Acknowledgements	73
74. Executive Summary	74
75. Abstract	75
76. Introduction	76
77. Objectives	77
78. Scope	78
79. Methodology	79
80. Results	80
81. Discussion	81
82. Conclusion	82
83. References	83
84. Appendix	84
85. Glossary	85
86. Bibliography	86
87. Index	87
88. Acknowledgements	88
89. Executive Summary	89
90. Abstract	90
91. Introduction	91
92. Objectives	92
93. Scope	93
94. Methodology	94
95. Results	95
96. Discussion	96
97. Conclusion	97
98. References	98
99. Appendix	99
100. Glossary	100
101. Bibliography	101
102. Index	102
103. Acknowledgements	103
104. Executive Summary	104
105. Abstract	105
106. Introduction	106
107. Objectives	107
108. Scope	108
109. Methodology	109
110. Results	110
111. Discussion	111
112. Conclusion	112
113. References	113
114. Appendix	114
115. Glossary	115
116. Bibliography	116
117. Index	117
118. Acknowledgements	118
119. Executive Summary	119
120. Abstract	120
121. Introduction	121
122. Objectives	122
123. Scope	123
124. Methodology	124
125. Results	125
126. Discussion	126
127. Conclusion	127
128. References	128
129. Appendix	129
130. Glossary	130
131. Bibliography	131
132. Index	132
133. Acknowledgements	133
134. Executive Summary	134
135. Abstract	135
136. Introduction	136
137. Objectives	137
138. Scope	138
139. Methodology	139
140. Results	140
141. Discussion	141
142. Conclusion	142
143. References	143
144. Appendix	144
145. Glossary	145
146. Bibliography	146
147. Index	147
148. Acknowledgements	148
149. Executive Summary	149
150. Abstract	150
151. Introduction	151
152. Objectives	152
153. Scope	153
154. Methodology	154
155. Results	155
156. Discussion	156
157. Conclusion	157
158. References	158
159. Appendix	159
160. Glossary	160
161. Bibliography	161
162. Index	162
163. Acknowledgements	163
164. Executive Summary	164
165. Abstract	165
166. Introduction	166
167. Objectives	167
168. Scope	168
169. Methodology	169
170. Results	170
171. Discussion	171
172. Conclusion	172
173. References	173
174. Appendix	174
175. Glossary	175
176. Bibliography	176
177. Index	177
178. Acknowledgements	178
179. Executive Summary	179
180. Abstract	180
181. Introduction	181
182. Objectives	182
183. Scope	183
184. Methodology	184
185. Results	185
186. Discussion	186
187. Conclusion	187
188. References	188
189. Appendix	189
190. Glossary	190
191. Bibliography	191
192. Index	192
193. Acknowledgements	193
194. Executive Summary	194
195. Abstract	195
196. Introduction	196
197. Objectives	197
198. Scope	198
199. Methodology	199
200. Results	200
201. Discussion	201
202. Conclusion	202
203. References	203
204. Appendix	204
205. Glossary	205
206. Bibliography	206
207. Index	207
208. Acknowledgements	208
209. Executive Summary	209
210. Abstract	210
211. Introduction	211
212. Objectives	212
213. Scope	213
214. Methodology	214
215. Results	215
216. Discussion	216
217. Conclusion	217
218. References	218
219. Appendix	219
220. Glossary	220
221. Bibliography	221
222. Index	222
223. Acknowledgements	223
224. Executive Summary	224
225. Abstract	225
226. Introduction	226
227. Objectives	227
228. Scope	228
229. Methodology	229
230. Results	230
231. Discussion	231
232. Conclusion	232
233. References	233
234. Appendix	234
235. Glossary	235
236. Bibliography	236
237. Index	237
238. Acknowledgements	238
239. Executive Summary	239
240. Abstract	240
241. Introduction	241
242. Objectives	242
243. Scope	243
244. Methodology	244
245. Results	245
246. Discussion	246
247. Conclusion	247
248. References	248
249. Appendix	249
250. Glossary	250
251. Bibliography	251
252. Index	252
253. Acknowledgements	253
254. Executive Summary	254
255. Abstract	255
256. Introduction	256
257. Objectives	257
258. Scope	258
259. Methodology	259
260. Results	260
261. Discussion	261
262. Conclusion	262
263. References	263
264. Appendix	264
265. Glossary	265
266. Bibliography	266
267. Index	267
268. Acknowledgements	268
269. Executive Summary	269
270. Abstract	270
271. Introduction	271
272. Objectives	272
273. Scope	273
274. Methodology	274
275. Results	275
276. Discussion	276
277. Conclusion	277
278. References	278
279. Appendix	279
280. Glossary	280
281. Bibliography	281
282. Index	282
283. Acknowledgements	283
284. Executive Summary	284
285. Abstract	285
286. Introduction	286
287. Objectives	287
288. Scope	288
289. Methodology	289
290. Results	290
291. Discussion	291
292. Conclusion	292
293. References	293
294. Appendix	294
295. Glossary	295
296. Bibliography	296
297. Index	297
298. Acknowledgements	298
299. Executive Summary	299
300. Abstract	300
301. Introduction	301
302. Objectives	302
303. Scope	303
304. Methodology	304
305. Results	305
306. Discussion	306
307. Conclusion	307
308. References	308
309. Appendix	309
310. Glossary	310
311. Bibliography	311
312. Index	312
313. Acknowledgements	313
314. Executive Summary	314
315. Abstract	315
316. Introduction	316
317. Objectives	317
318. Scope	318
319. Methodology	319
320. Results	320
321. Discussion	321
322. Conclusion	322
323. References	323
324. Appendix	324
325. Glossary	325
326. Bibliography	326
327. Index	327
328. Acknowledgements	328
329. Executive Summary	329
330. Abstract	330
331. Introduction	331
332. Objectives	332
333. Scope	333
334. Methodology	334
335. Results	335
336. Discussion	336
337. Conclusion	337
338. References	338
339. Appendix	339
340. Glossary	340
341. Bibliography	341
342. Index	342
343. Acknowledgements	343
344. Executive Summary	344
345. Abstract	345
346. Introduction	346
347. Objectives	347
348. Scope	348
349. Methodology	349
350. Results	350
351. Discussion	351
352. Conclusion	352
353. References	353
354. Appendix	354
355. Glossary	355
356. Bibliography	356
357. Index	357
358. Acknowledgements	358
359. Executive Summary	359
360. Abstract	360
361. Introduction	361
362. Objectives	362
363. Scope	363
364. Methodology	364
365. Results	365
366. Discussion	366
367. Conclusion	367
368. References	368
369. Appendix	369
370. Glossary	370
371. Bibliography	371
372. Index	372
373. Acknowledgements	373
374. Executive Summary	374
375. Abstract	375
376. Introduction	376
377. Objectives	377
378. Scope	378
379. Methodology	379
380. Results	380
381. Discussion	381
382. Conclusion	382
383. References	383
384. Appendix	384
385. Glossary	385
386. Bibliography	386
387. Index	387
388. Acknowledgements	388
389. Executive Summary	389
390. Abstract	390
391. Introduction	391
392. Objectives	392
393. Scope	393
394. Methodology	394
395. Results	395
396. Discussion	396
397. Conclusion	397
398. References	398
399. Appendix	399
400. Glossary	400
401. Bibliography	401
402. Index	402
403. Acknowledgements	403
404. Executive Summary	404
405. Abstract	405
406. Introduction	406
407. Objectives	407
408. Scope	408
409. Methodology	409
410. Results	410
411. Discussion	411
412. Conclusion	412
413. References	413
414. Appendix	414
415. Glossary	415
416. Bibliography	416
417. Index	417
418. Acknowledgements	418
419. Executive Summary	419
420. Abstract	420
421. Introduction	421
422. Objectives	422
423. Scope	423
424. Methodology	424
425. Results	425
426. Discussion	426
427. Conclusion	427
428. References	428
429. Appendix	429
430. Glossary	430
431. Bibliography	431
432. Index	432
433. Acknowledgements	433
434. Executive Summary	434
435. Abstract	435
436. Introduction	436
437. Objectives	437
438. Scope	438
439. Methodology	439
440. Results	440
441. Discussion	441
442. Conclusion	442
443. References	443
444. Appendix	444
445. Glossary	445
446. Bibliography	446
447. Index	447
448. Acknowledgements	448
449. Executive Summary	449
450. Abstract	450
451. Introduction	451
452. Objectives	452
453. Scope	453
454. Methodology	454
455. Results	455
456. Discussion	456

CHAPTER I

INTRODUCTION

Summary

In recent years the measurement and control of phase has become of increasing importance to communications engineering. This is particularly true in the military fields where equipments such as monopulse radar, guided missiles, radio navigational systems, and sonar gears for both submarines and surface ships have become highly developed. Many important non-military applications also exist, among which are the outphasing system of single sideband radio telephony and certain systems of commercial aircraft radio navigation.

It has been observed that most texts on radio measurements and radio engineering treat the subject of phase measurement at the best in a prefuctionary manner. There is, however, a great deal of information available in the literature. This paper proposes to present a consolidated discourse on available information on the measurement and control of phase. The basic intent is to provide the student or engineer with enough of the theoretical and practical information on the subject to resolve most problems encountered.

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It was originally intended to extend the topic of phase measurement to include the microwave region. When it becomes apparent that the topic was going to be far too broad for a paper of this scope it was restricted to cover only circuits with lumped constants. Strict classification is to be expected also in connection with certain equipments

operating in the microwave region by the nature of their operational uses. Unfortunately this often acts to limit discussion on basic principles which in themselves are unclassified; this was another reason for excluding microwave circuits.

A great deal of first hand information on the practical aspects of phase handling circuits was obtained during a ten week industrial tour with the Pacific Division of the Bendix Aviation Corporation at Los Angeles, California. During this time numerous occasions arose in which a simple four terminal device which would accurately and directly measure the phase relationships between two voltages would have been welcomed.

There are three general approaches to the problem of phase measurement: 1) the visual display of the signal and reference waveforms thereby obtaining the phase relationship by means of physical measurements applied to these waveforms, 2) the conversion of phase difference into an analog of current or voltage, and 3) the insertion of a calibrated phase shift in series with the signal to bring it into phase coincidence with the reference as noted by some phase null indicator.

The visual methods known are restricted to single frequency sinusoidal signals. The concepts involved and equipment required are relatively simple and the accuracy obtained is not good except in certain special cases.

Electronic phasemeters are not in general use. In fact, the few reasonably accurate phasemeters known are complex and bulky enough to be called stationary. All electronic phasemeters must perform one or more of the following functions:

According to the statement made by the witness in his deposition...

...the witness further stated that he had been present...

...and that he had been present at the same time...

...and that he had been present at the same time...

...and that he had been present at the same time...

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...and that he had been present at the same time...

...and that he had been present at the same time...

1. Remove amplitude variations from reference and measured signal.
2. Shift the phase of either or both reference and measured signal by some fixed amount.
3. Provide for null indication at the condition of phase coincidence or be equipped with a discriminator circuit which responds to magnitude and sense of phase difference.

Calibrated phase shifters are obtainable and capable of very accurate single frequency performance. Like the visual methods they are of no use for complex voltages. Also the frequency division required by some of these shifters is troublesome. Their most useful applications are for use as standards and in systems having a very carefully controlled single frequency signal. The most important precision phase shifter is the goniometer, widely in use in the field of radio navigation.

Going back to the considerations of the electronic phasemeter, in the last few years a tremendous amount of work has been done in fields of phase shift networks and phase discriminators. It is the attempt of this paper to present the most important data on these subjects.

Amplitude limiting circuits have been in use for many years with few modifications since their applications are not numerous. Recently a very simple limiter circuit, the cathode coupled clipper, has been proposed which promises to be of importance in speed transmission systems and any other application where limiting is required up to one megacycle. Because of its simplicity and apparent superiority to any other circuit for phasemeter limiting operations it will be considered in some detail.

1. The first thing I noticed when I stepped out of the plane was the fresh air. It felt like I had been in a bubble for the last few days. The sun was shining brightly, and the birds were singing. It was a beautiful sight.

2. The second thing I noticed was the smell of the flowers. They were in full bloom, and the fragrance was everywhere. It was a pleasant surprise, and it made me feel like I had found a hidden gem.

3. The third thing I noticed was the sound of the water. It was a gentle lapping against the shore, and it was so soothing. I had never heard it before, and it felt like a lullaby.

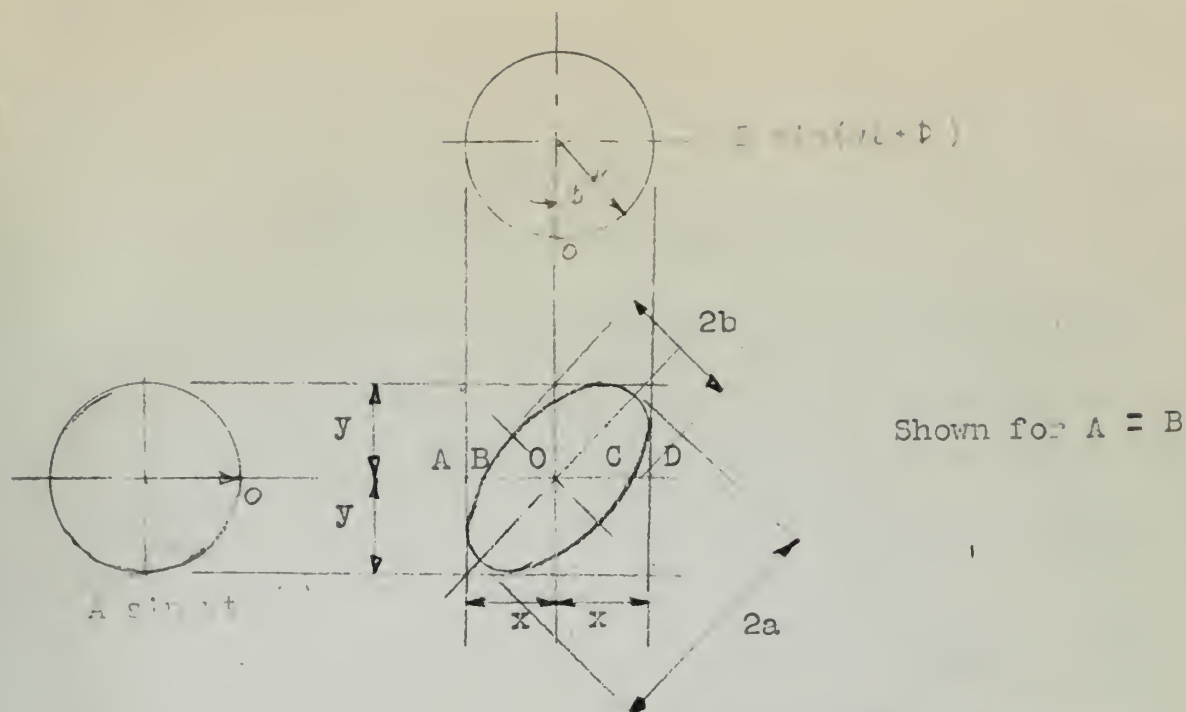
4. The fourth thing I noticed was the taste of the food. It was a delicious surprise, and it was exactly what I needed. I had been traveling for days, and I was starving.

5. The fifth thing I noticed was the feeling of the sun on my face. It was warm and comforting, and it was exactly what I needed. I had been feeling a bit down, and it felt like a ray of light.

6. The sixth thing I noticed was the sight of the mountains. They were majestic and beautiful, and they were exactly what I needed. I had been looking for a place to relax, and I had found it.

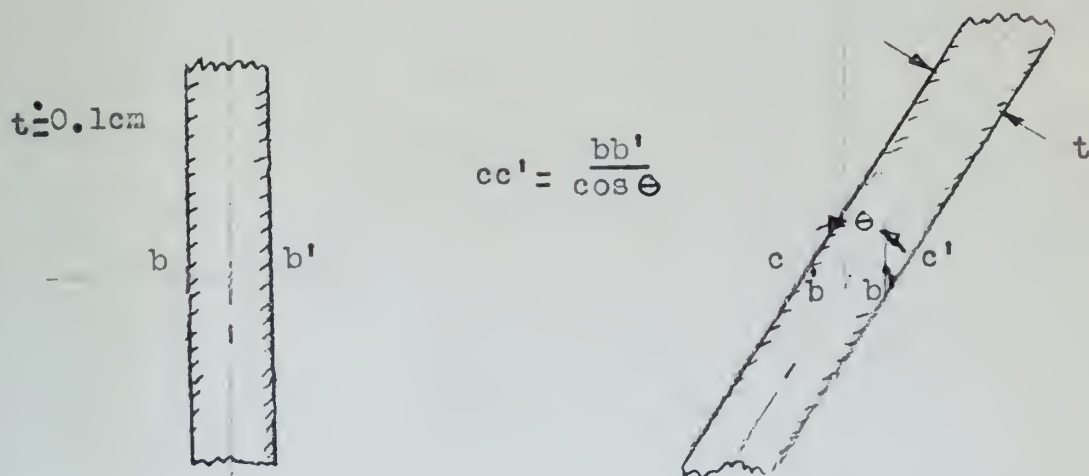
7. The seventh thing I noticed was the sound of the wind. It was a gentle breeze, and it was so refreshing. I had never felt it before, and it felt like a breath of fresh air.

8. The eighth thing I noticed was the feeling of the sand under my feet. It was soft and warm, and it was exactly what I needed. I had been feeling a bit tired, and it felt like a warm blanket.



Lissajoux Phase Pattern

Figure 1



Observation Error due to
Trace Thickness

Figure 4

CHAPTER II

VISUAL METHODS OF PHASE MEASUREMENT

The visual method of presentation and comparison of electrical signals for phase measurement is perhaps the most universally used of any method possible. Contrary to first impression it is not inherently inaccurate and in certain cases it may prove to be the most accurate method possible, either practically or theoretically.

The familiar Lissajoux figure is illustrated in Figure 1. The following relationships are obtained from the figure

$$\begin{aligned} BOC &= 2B \sin(\omega t + \phi) / \omega t = 0 = 2B \sin \phi \\ AOD &= 2B \quad \frac{BOC}{AOD} = \sin \phi \end{aligned} \quad (1-1)$$

Obviously the method is most accurate at $\phi = n\pi$ where the ellipse degenerates into a straight line. In this case extremely accurate comparison is possible (Appendix I). This method is used again in the discussion of phase standards in a later chapter.

There are two principal sources of error when the phase angle is not a multiple of π and the sine is obtained by intercept ratio. There are 1) observer error due to the CRO trace having appreciable thickness and 2) error due to deviation from an elliptical pattern with the presence of harmonics in the signal. Extensive mathematical treatment has been given to these considerations¹ and the theoretical results are shown in Figures 2 and 3. The mathematical treatment is quite long and, because of the nature of some of the assumptions used,

THEORY OF THE EARTH

The first part of the theory is devoted to the study of the earth's internal structure. It is shown that the earth is composed of several layers, the outermost of which is the crust. The crust is divided into two parts, the upper and the lower. The upper part is the continental crust, and the lower part is the oceanic crust. The continental crust is thicker than the oceanic crust, and it is composed of a variety of rocks, including granite, gneiss, and schist. The oceanic crust is thinner than the continental crust, and it is composed of a variety of rocks, including basalt, gabbro, and diorite.

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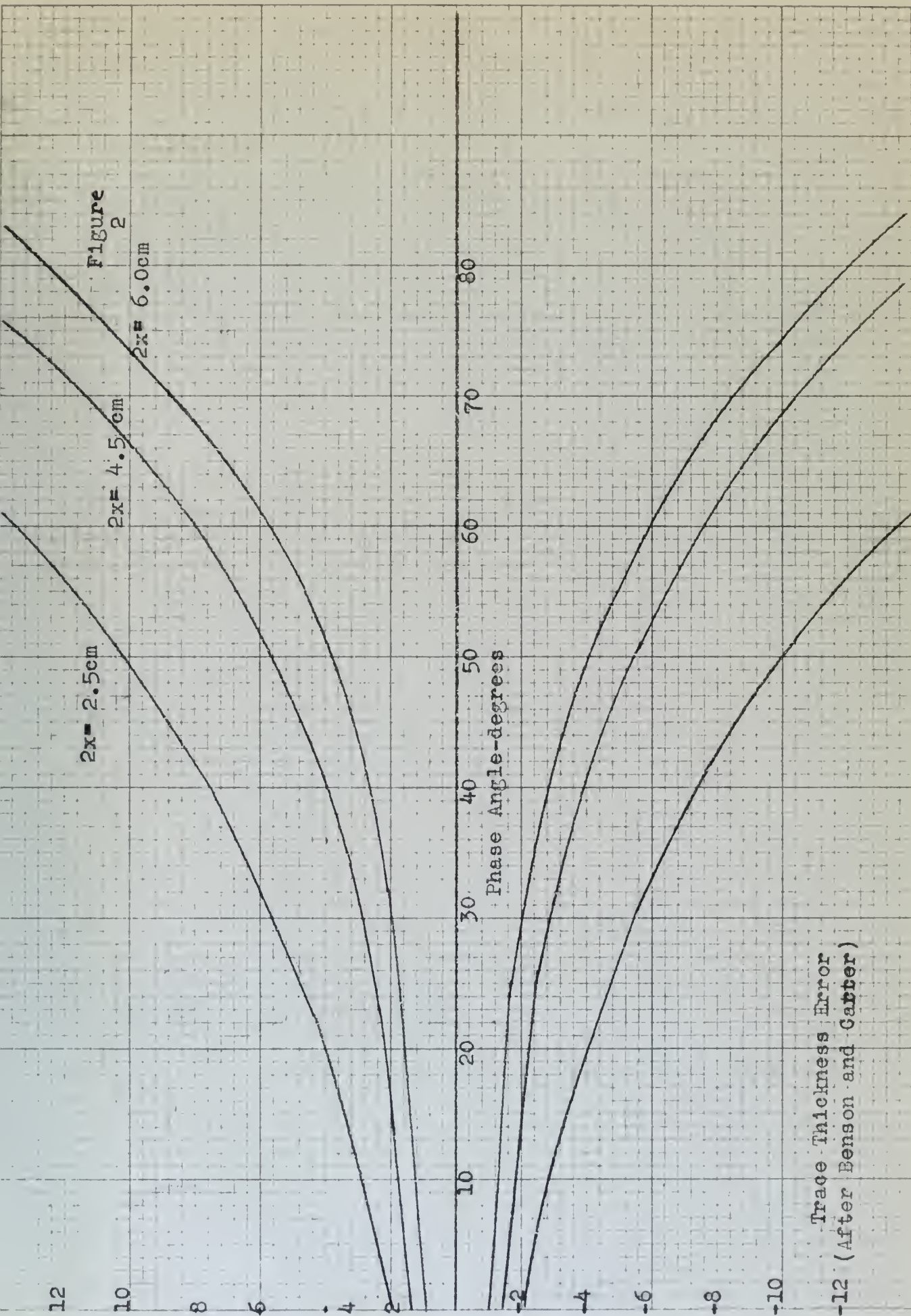
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$$(1-1) \quad \frac{d}{dt} \left(\frac{1}{r} \right) = - \frac{1}{r^2} \frac{dr}{dt}$$

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Trace Thickness Error
-12 (After Benson and Carter)

Figure
2

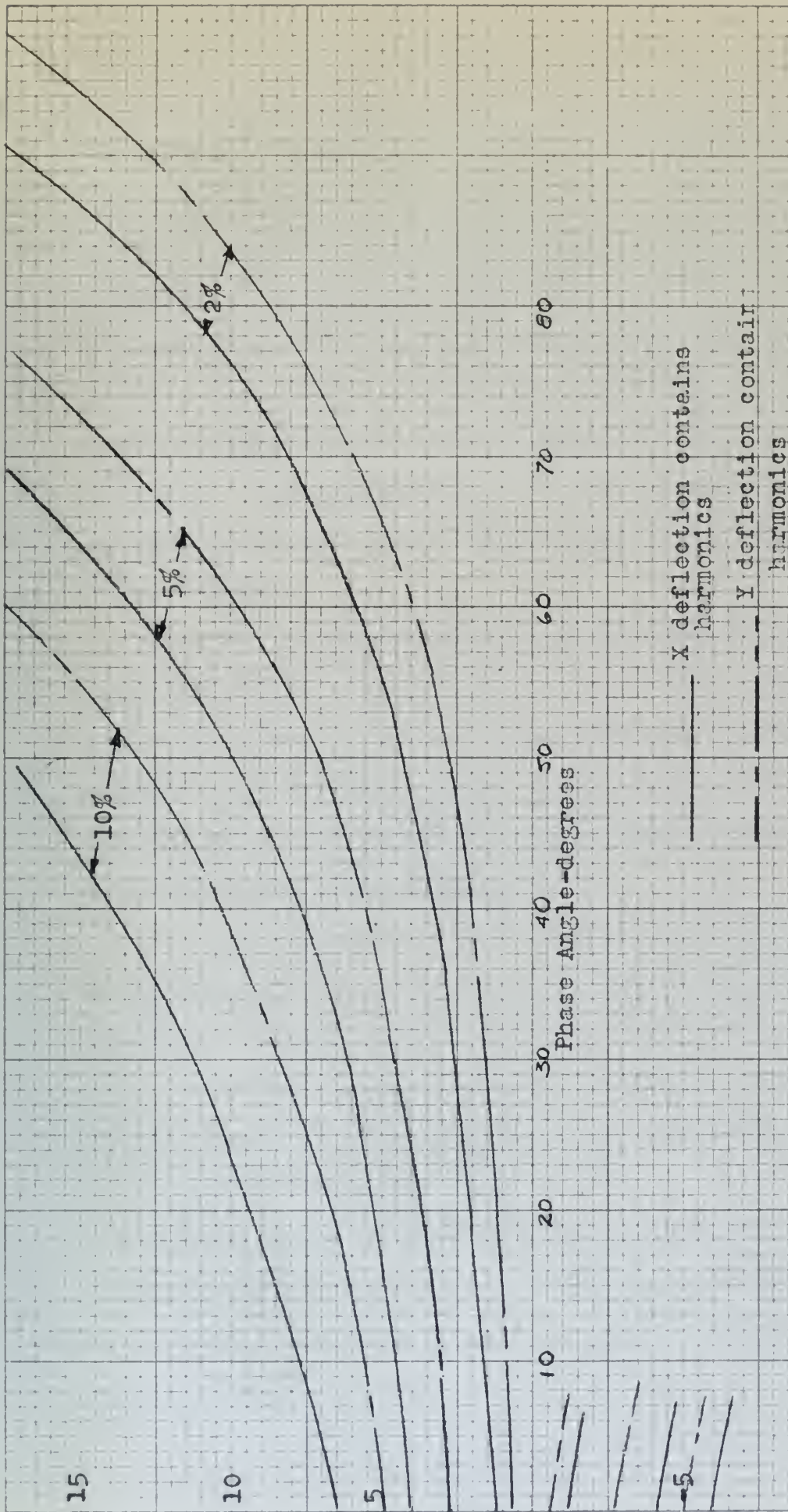


Figure 3

Phase Angle Measurement Error due to Harmonic Content
(After Benson and Carter)

was adjudged to be of too little value to be included as an appendix. The assumptions made are that the trace is a constant thickness of 1mm. and that the observer is consistently able to read the intercept within an accuracy of plus or minus 0.5mm. Then, as the angle of transit of the trace across the reference axis changes with phase angle, the effective intercept as measured also changes as is illustrated in Figure 4. This results in the theoretical error becoming a transcendental function of phase angle. Experimental results show the agreement between theory and actual measurement to be no greater than 50% at phase angles between 30 and 60 degrees.

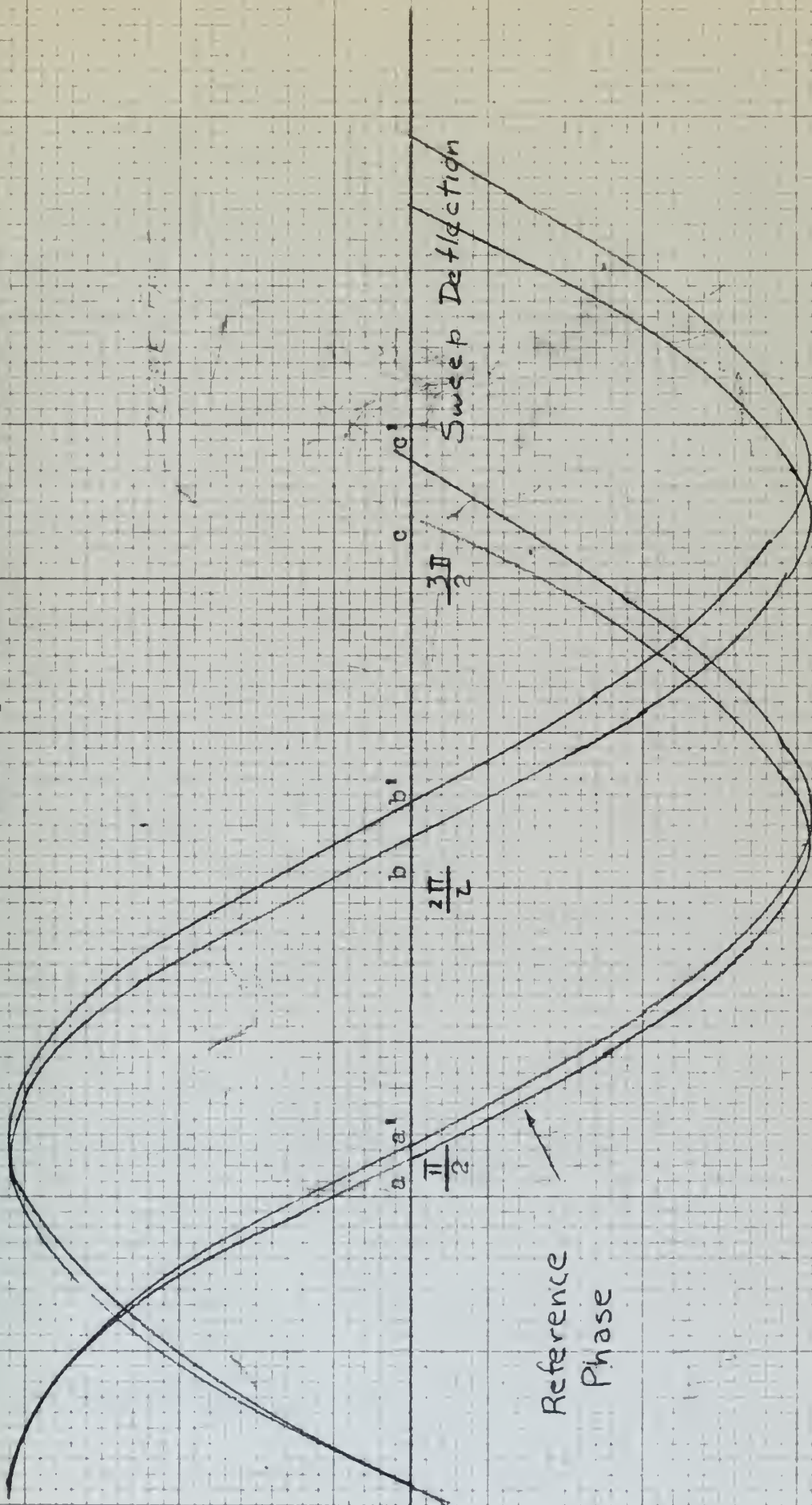
In summary of this method it may be said that with great care in making observations with a 6 inch effective diameter CRO and applying corrections, phase angles up to 45-60 degrees can be measured with an accuracy of plus or minus 1 degree. In the special case where $\phi = n\pi$ the error can become negligible and phase angles as small as 0.3 degree can be determined (Appendix I).

A second visual method which is frequently used is simultaneous visual observation of two signals by means of a dual beam CRO or single gun CRO and electronic switch. The first method is to be preferred since the several electronic switches examined have a lower limit of switching frequency which limits the minimum frequency of the signal to be examined to about 100 cycles.

Possible sources of error in measuring phase with a dual beam CRO are 1) wave distortion due to non-linear sweep, 2) amplitude distortion in the vertical amplifier, 3) observer error in locating identical phase points.

[illegible]

Effect of Non-linear Sweep on the Intercept Method of Phase Measurement



Reference
Phase

Sweep Deflection

The second of these can be easily avoided by choosing the reference points as the intersection of the trace with the zero deflection axis as established by grounding the input terminals. Clearly the steeper the angle of intercept of the trace with this reference axis the smaller the observer error. This had been previously brought out in the discussion of the error due to trace thickness in the Lissajoux pattern method. Therefore, by using maximum deflection and carefully locating the axis of zero deflection two of the error sources can be minimized. There remains only the sweep nonlinearity to be considered.

Assume the two signals to be compared are sinusoidal and are displayed on a dual beam CRO so adjusted to place both traces on a common axis. For a given signal frequency and sweep length the sweep nonlinearity can be established as a function of the electrical angle θ , $f(\theta)$ say, Figure 5. Further assume that the value of the phase angle is established by the relationship

$$\phi' = \frac{ab'}{ac'} \times \pi \quad (1-2)$$

with the sweep so controlled to display one half cycle of the reference phase across the scope face. This is the maximum degree of expansion possible.

If the sweep were exactly linear the true phase angle would be given in terms of the undistorted intercepts

$$\phi = \frac{ab}{ac} \times \pi$$

since

$$\begin{aligned} ab' &= ab + f(\theta_1) = ab + f(\phi) \\ ac' &= ac + f(\theta_2) = ab + f(\pi) \end{aligned}$$

and the error in measurement of the phase angle is

and would of course be likely to be of considerable importance in the determination of the rate of the reaction. It is therefore of interest to determine the rate constants for the reaction of the radical with the gas. The rate constants for the reaction of the radical with the gas have been determined by the method of the initial rates. The rate constants for the reaction of the radical with the gas have been determined by the method of the initial rates. The rate constants for the reaction of the radical with the gas have been determined by the method of the initial rates. The rate constants for the reaction of the radical with the gas have been determined by the method of the initial rates.

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$$k = \frac{1}{t} \ln \frac{a}{a-x}$$

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It is found that the rate constants for the reaction of the radical with the gas are of the order of 10^4 to 10^5 l./mole-sec.

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$$k = \frac{1}{t} \ln \frac{a}{a-x}$$

where

$$k = \text{rate constant for the reaction of the radical with the gas}$$

$$t = \text{time in seconds}$$

$$a = \text{initial concentration of the radical}$$

$$x = \text{concentration of the radical at time } t$$

and the rate of reaction of the radical with the gas is

$$\epsilon = \phi' - \phi$$

in terms of the constants of the system

$$\begin{aligned}\epsilon &= \pi \left\{ \frac{ab + f(\phi)}{ac + f(\pi)} - \frac{ab}{ac} \right\} \\ &= \pi \left\{ \frac{f(\phi) - \frac{ab}{ac} f(\pi)}{ac + f(\pi)} \right\} \\ &= \frac{\pi f(\phi) - \phi f(\pi)}{ac + f(\pi)}\end{aligned}\quad (1-3)$$

To determine at what value of ϕ the error will be a maximum or minimum differentiate the function with respect to

$$\frac{d\epsilon}{d\phi} = \frac{\pi \frac{df(\phi)}{d\phi} - f(\pi)}{ac + f(\pi)} \quad (1-4)$$

The simplest sweep deflection circuit is a sawtooth generator using the first portion of an exponentially varying voltage where the exponential is almost linear in form. In this case the equation for trace deflection across the scope becomes

$$x = B(1 - e^{-t/T}) \quad (1-5)$$

where B is some maximum deflection and T is the time constant of the system. Since, for any given electrical frequency and signal presentation, the physical deflection can be related to the electrical angle by

$$t = \theta/\omega$$

and

$$\begin{aligned}f(\theta) &= B(1 - e^{-\theta/T\omega}) - A\theta \\ \epsilon &= \frac{\pi [B(1 - e^{-\phi/T\omega}) - A\phi] - \phi [B(1 - e^{-\pi/T\omega}) - A\pi]}{ac + B(1 - e^{-\pi/T\omega}) - A\pi}\end{aligned}$$

Now

$$\frac{d\epsilon}{d\phi} = \frac{\frac{B\pi}{T\omega} (e^{-\phi/T\omega}) - B(1 - e^{-\pi/T\omega})}{ac + B(1 - e^{-\pi/T\omega}) - A\pi}$$

This gives the value of ϕ at which the error in measurement is a maximum to be

$$d\mathbf{r} = \frac{1}{2} d\mathbf{r} + \frac{1}{2} d\mathbf{r}$$

where the first term is the derivative of the function

$$\left\{ \frac{1}{2} \frac{d\mathbf{r}}{dt} - \frac{1}{2} \frac{d\mathbf{r}}{dt} \right\} = \frac{1}{2} \frac{d\mathbf{r}}{dt}$$

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$$\frac{1}{2} \frac{d\mathbf{r}}{dt} = \frac{1}{2} \frac{d\mathbf{r}}{dt}$$

(1-1)

where the second term is the derivative of the function

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$$\frac{1}{2} \frac{d\mathbf{r}}{dt} = \frac{1}{2} \frac{d\mathbf{r}}{dt} = \frac{1}{2} \frac{d\mathbf{r}}{dt}$$

(1-2)

where the third term is the derivative of the function

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$$\frac{1}{2} \frac{d\mathbf{r}}{dt} = \frac{1}{2} \frac{d\mathbf{r}}{dt} = \frac{1}{2} \frac{d\mathbf{r}}{dt}$$

(1-3)

where the fourth term is the derivative of the function

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$$\frac{1}{2} \frac{d\mathbf{r}}{dt} = \frac{1}{2} \frac{d\mathbf{r}}{dt} = \frac{1}{2} \frac{d\mathbf{r}}{dt}$$

(1-4)

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(1-5)

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where the fifth term is the derivative of the function

(1-6)

$$\phi = Tw \ln \left[\frac{\pi}{Tw(1 - e^{-\pi/Tw})} \right] \quad (1-6)$$

This function is indeterminate in the limits, but investigation in the region $Tw = 1$ to $Tw = 100$ indicates ϕ of maximum error to be π radians when $Tw = 30$. The interpretation of this is that for $Tw = 30$ the error is approximately proportional to the value of ϕ and for Tw greater than 30 the sweep approaches linearity and the error goes to zero. For values of Tw less than 30, ϕ of maximum error approaches zero.

A minimum condition arises when $f(\theta) = K\theta$ which can be verified by substitution into the error equation 1-3

$$\epsilon = \frac{\pi k \phi - \phi k r}{ac + k r} = 0$$

This is a trivial case since

$$ab' = ab + k\phi \quad \& \quad ac' = ac + k r$$

represent another linear sweep of a different rate.

The implications are that this method is inherently more accurate for larger values of phase angle, at least with this particular type of sweep distortion which seems the most likely.

These same expressions can be used to estimate the probable measurement error due to observational error by setting

$$ab' = ab \pm k \quad ; \quad ac' = ac \pm k$$

giving

$$\epsilon = \pm \left\{ \phi \left(\frac{k}{ac+k} \right) - \left(\frac{k r}{ac+k} \right) \right\}$$

This represents the most prevalent type of human error where the observer's measurements are consistently too large or too small. Since very linear sweeps are possible in well made oscilloscopes, it is concluded that the

$$(8-1) \quad \int_{-\infty}^{\infty} f(x) \delta(x-a) dx = f(a)$$

This equation is interpreted as follows: the integral of the function $f(x)$ multiplied by the Dirac delta function $\delta(x-a)$ is equal to the value of the function $f(x)$ at $x=a$. The Dirac delta function is a generalized function which is zero everywhere except at $x=a$, where it is infinite. It is defined by the equation

$$\int_{-\infty}^{\infty} \delta(x-a) f(x) dx = f(a)$$

$$\delta(x-a) = \lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} \delta_{\epsilon}(x-a)$$

where $\delta_{\epsilon}(x-a)$ is a function which is zero everywhere except in the interval $(a-\epsilon, a+\epsilon)$, where it is equal to $1/\epsilon$. The Dirac delta function is a useful tool in many areas of physics and engineering, particularly in the study of point charges and point masses.

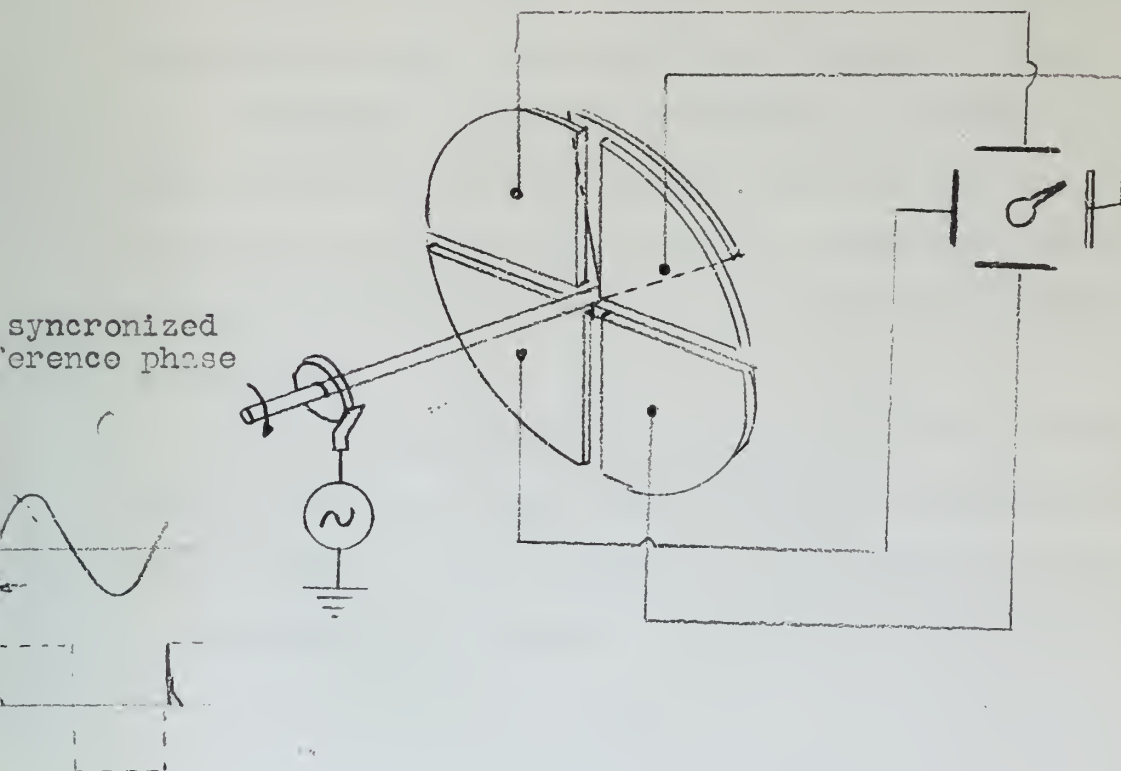
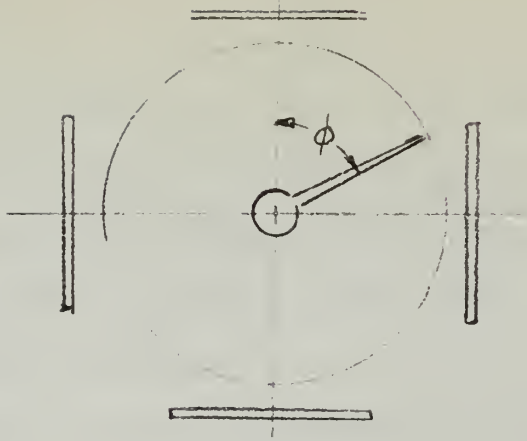
There are many other properties of the Dirac delta function, which can be derived from the defining equation. For example, it is easy to show that

$$\int_{-\infty}^{\infty} \delta(x-a) dx = 1$$

This is because the Dirac delta function is a generalized function which is zero everywhere except at $x=a$, where it is infinite. It is defined by the equation

Visual Phase Measurement by Circular Trace Presentation

Figure 6



theoretical accuracy obtainable by the use of this method is of the order of magnitude

$$\epsilon \leq \frac{k \pi}{a_c + k}$$

If a 6 inch usable scope face and an observer error of plus or minus 0.5 mm are chosen the error becomes 1.16 degrees maximum which is good enough for most work. Having the entire range of 0-360 displayed unambiguously is also an advantage of this method. Obviously the larger the CRO the smaller the error.

A variation of this basic method can be used with a time based sweep single gun oscilloscope when provisions can be made to trigger the sweep externally. A typical method would be as follows: the reference signal is squared, differentiated and the negative peak clipped. The spike voltage thus obtained is used to trigger the sweep upon which the signal to be measured is displayed. By use of time markers or intercept measuring technique similar to that previously described, the phase difference may be determined. The obvious disadvantage of this method is that the trigger signal does not pass through an identical channel as the signal to be measured and is not of the same form; phase shift inherent in the CRO amplifiers may manifest itself as an apparent phase difference between the two signals.

A third general approach to the display by CRO is by obtaining a circular trace which is then either Z-modulated or momentarily expanded to produce "spikes" as shown in Figure 6. The circular sweep may be obtained by splitting one of the signals in quadrature by an R-C network, or it

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$$\frac{a+b}{c+d} = \frac{e}{f}$$

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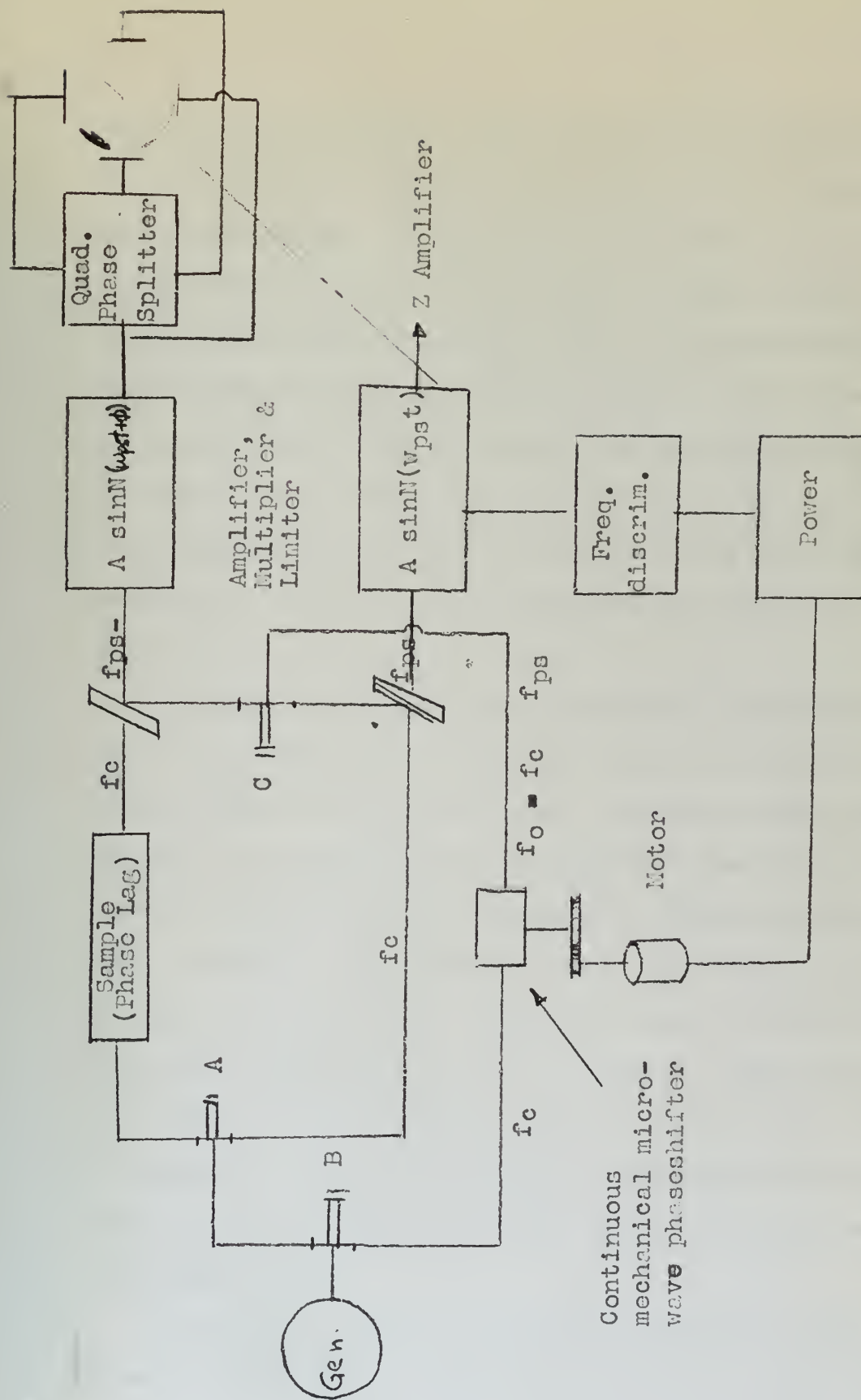


Figure 7

A Laboratory Application of Z-Modulation
Phase Measurement

can be obtained externally. These methods may be found useful when there is the possibility of multiplying the frequency by a large factor thereby multiplying the phase difference by a like factor.

An experimental device²⁰ developed to measure the index of refraction of air by measurement of the phase shift of an electro-magnetic wave passed through the medium is shown in Figure 7. This device uses Z-modulation of a CRO to indicate the shift. By frequency multiplication of 1875:1 very small shifts can be measured. This large shift is also necessitated by the fact that the signal is applied directly to the Z amplifier as a sine wave and the modulated beam appears as an arc rather than a spot.

Referring to the figure the signal source is a klystron operating in the region of 400 mcs. Magic Tees A, B, and C distribute the r.f. energy through the three paths shown. The signal passing through the sample of air under examination is retarded in some amount. The continuously rotating mechanical phaseshifter is driven by a source synchronized to klystron. This continuous phase shift changes the frequency by slightly more than 100 cps. This shifted signal is used as a local oscillator to heterodyne down both the reference signal and the signal to be examined for phase shift to a useable audio frequency. The two audio frequencies are multiplied and limited, then applied to the phase measuring circuit in this case a CRO. Crystal mixers are used in the heterodyning process.

This system is cited as a practical application of phase measuring technique, also because it illustrates the fact that phase difference can

be multiplied (or divided) by multiplying frequency, and that phase relationships remain invariant during heterodyning. The latter is not quite so obvious as the former. In the interest of complete treatment of the subject proof of this is shown in Appendix II.

The implications of these two properties are thought to hold considerable importance to the problem of phase measurement. At the expense of suitable circuitry for frequency multiplication or division any degree of accuracy desired can be obtained. Also, when the space considerations predominate, a local oscillator and lumped constant discriminator may provide space savings over a microwave discriminator. In any event, it puts the upper limit of frequency for any satisfactory phase meter at that frequency which can be heterodyned into the useful range of the instrument.

R-C Lattice Phaseshift Network (Dome Network)

Figure 8

$$RC_{11} = RC_{22} = RC_{33} \quad C_2 = aC_1 \quad R_2 = \frac{R_1}{a}$$

$$R_3 = \left[\frac{1-4a}{4a} \right] R_2 \quad C_3 = \left[\frac{4a^2}{1-4a} \right] C_1$$

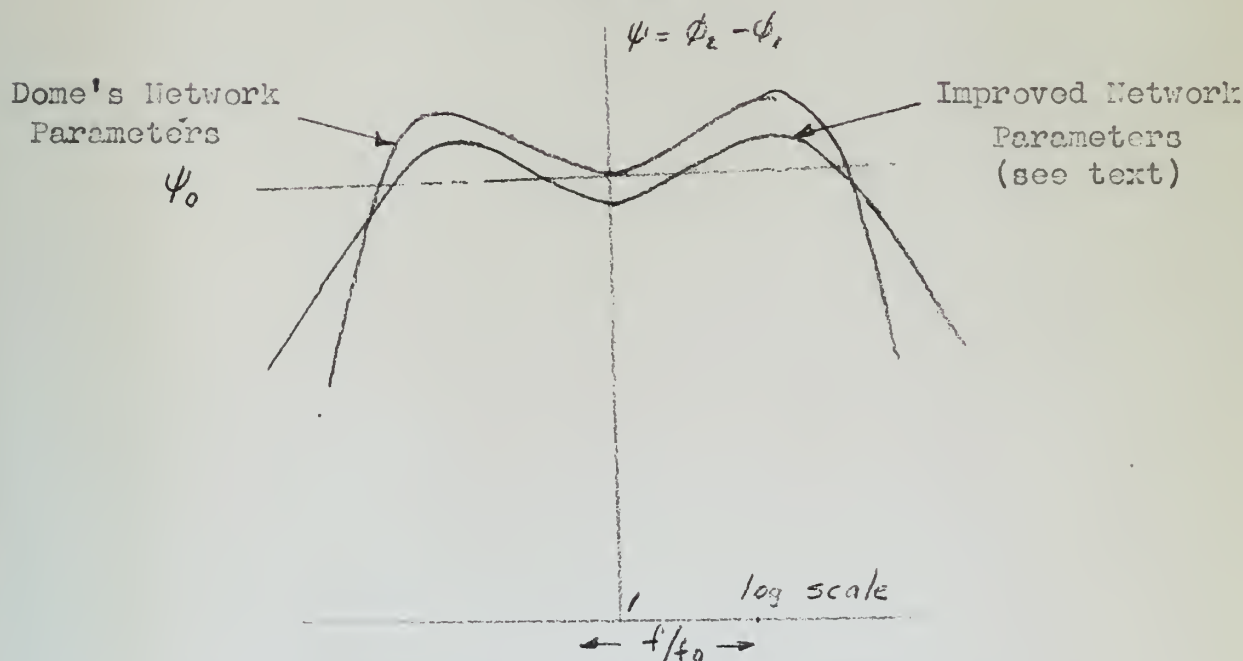
$$f_0 = \frac{1}{2\pi R_1 C_1} \quad a = \frac{1}{s+2}$$

$$s \geq 2 \quad s = \frac{1-2a}{a}$$

$$|e_0| = (1-4a)|e| = \frac{(s-2)}{(s+2)} |e|$$

$$\phi = \tan^{-1} \frac{2sff_0(f^2 - f_0^2)}{(f - f_0) - sf^2 f_0}$$

Figure 9



Output Phase Characteristic for a Pair of Networks
Used to Provide a Constant Phase Difference over a
Wide Band of Frequencies

CHAPTER III

CONSIDERATIONS OF AN ELECTRONIC PHASEMETER

As previously mentioned an electronic phasemeter may perform any or all of these three functions 1) phase shift of a predetermined amount in either or both channels, 2) removal of amplitude variations in the signals, 3) discrimination of the phase to produce an analog output of current or voltage.

1. Phase Shifting

When the phase shift desired is 180 degrees it may be obtained at once by a vacuum tube inverter or by a transformer. When the shift desired is something other than 180 degrees complications arise. It is not hard to devise a circuit to create a continuously varying output phase, but these circuits generally have the unpleasant quality of a continuously varying output amplitude. The consideration of a passive, constant attenuation, broadband phase network has received considerable attention in recent years particularly in view of its applications to single sideband radio transmission. Because of its inherent importance to the problem of broadband phase measurement, it is not considered too much of a digression to treat these networks in some detail here.

The first intensive treatment of such a network was made by Dome⁴. He proposed the half lattice R-C network illustrated in Figure 8, and developed the necessary relationships between circuit parameters. Described briefly the circuit exhibits a phase characteristic which is proportional to the logarithm of the signal frequency. When two of these

THE HISTORY OF THE UNITED STATES

The history of the United States is a story of growth and change. It is a story of a people who have built a great nation from a small colony. The story begins with the first settlers who came to the New World in search of a better life. They found a land of opportunity and freedom, and they built a nation that has become a model for the world. The story continues through the years of struggle and triumph, from the American Revolution to the present day. It is a story of a people who have never been satisfied with the status quo, and who have always been striving for a better future.

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networks are used in conjunction the difference in the phase angles of their output voltages may be held constant over a considerable range of frequency.

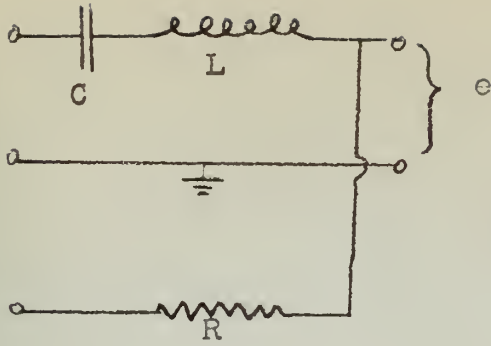
$$\begin{aligned}
 \phi_1 &= \ln f + C \\
 \phi_2 &= \ln kf + C \\
 \phi_2 - \phi_1 &= \ln kf - \ln f \\
 &= \ln k \\
 &= \text{Constant}
 \end{aligned}
 \tag{2-1}$$

When using a pair of these networks to obtain a constant phase difference Dome proposed that at f_0 , the geometrical mean frequency of the required operating frequency band, the respective phase shifts of the two networks be 180 degrees plus or minus 45 degrees, assuming the phase constant to be 90 degrees for the purposes of illustration. A careful plot of the phase difference between the networks shows the maximum deviation from the required difference to occur near the upper and lower limits of the frequency band, Figure 9. Obviously if the maximum phase excursion is not qualified by a sense restriction there is no reason to restrict the phase difference error to zero at the mean frequency. Laden in an unpublished thesis at the U. S. Naval Postgraduate School¹¹ attacked the problem from the standpoint of the theory of a function of a real variable. By manipulation of the circuit equations he obtained the equation

$$\phi_\beta - \phi_\alpha - 90^\circ = 2 \left(\tan^{-1} \frac{\zeta_\beta \omega \omega_{0\beta}}{\omega^2 - \omega_{0\beta}^2} - \tan^{-1} \frac{\zeta_\alpha \omega \omega_{0\alpha}}{\omega^2 - \omega_{0\alpha}^2} - 45^\circ \right)
 \tag{2-2}$$

which bears a marked similarity to Dome's equation. By treating this function to produce a minimum maximum the following relationships between

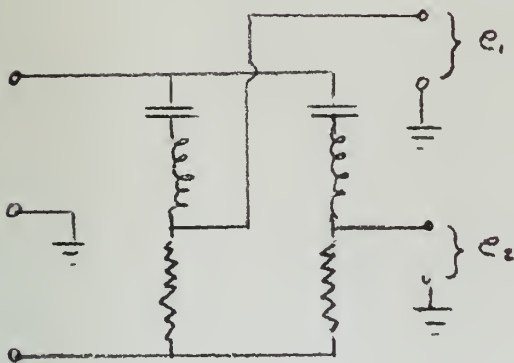
A Basic L-C-R Constant Attenuation Phase Shift Network



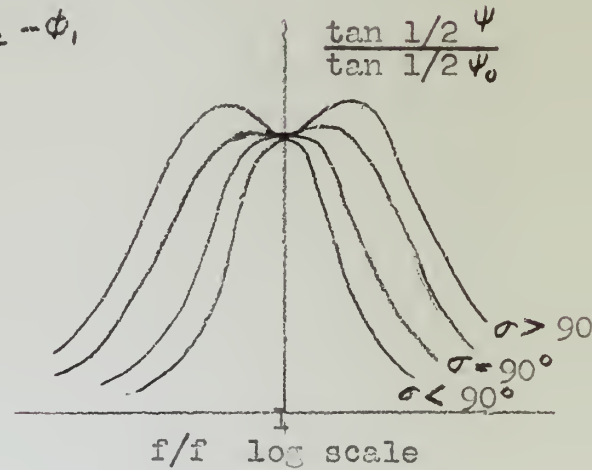
$$\begin{aligned} \omega_1^2 &= 1/LC \\ Q^2 &= L/CR \\ k &= 1/2 \\ L &= QR/\omega_1 \\ C &= 1/\omega_1 QR \end{aligned}$$

Figure 11

Characteristics of a Constant Phase
Difference Network
(After Luck)



$$\psi = \phi_2 - \phi_1$$



circuit parameters are found to hold.

$$\omega_0 \alpha = g - \alpha$$

$$\omega_0 \beta = h - \alpha$$

$$\alpha = \sqrt{\omega(\text{upper f.}) \omega(\text{lower f.})}$$

$$\rho = \sqrt{\frac{\omega(\text{upper f.})}{\omega(\text{lower f.})}}$$

$$u = h - g$$

$$h = 1/g$$

$$u = \sqrt{(p+1) [\sqrt{p^2+1} + \sqrt{2} p - (p+1)]/p}$$

These relationships enable one at once to choose the optimum circuit parameters having decided upon the band width.

An article by Luck which appeared at the same time gives a much clearer picture of the problem since he arrives at exactly the same results as Laden with none of the ponderous mathematical treatment¹². Luck's results must be used in graphical form and do not provide an analytical expression for circuit parameters. Starting with the simple L-C-R lattice of Figure 10 Luck developed an expression for the complex ratio of output to input voltage and used this expression as a basis of all subsequent development. The output voltage of the circuit of Figure 10 is

$$e = -\frac{E}{Z} + \frac{R E}{R + j\omega L + 1/j\omega C}$$

using the conventional expression for the quality of a circuit

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 R C} \quad (\omega_0 \text{ is resonant freq.})$$

and dividing through by R and simplifying the resultant expression for voltage transfer becomes

$$\frac{e}{E} = K \frac{1 + jQ(f/f - f/f_1)}{1 - jQ(f/f - f/f_1)} \quad (2-3)$$

To evaluate the phase angle between input and output voltages let

$$Q(f/f - f/f_1) = \tan 1/2 \phi$$

Several parameters are known to date.

$$\begin{aligned} \frac{1}{\sqrt{1-\beta^2}} \frac{d\beta}{dt} &= \frac{1}{\sqrt{1-\beta^2}} \frac{d}{dt} \left(\frac{v}{c} \right) = \frac{1}{\sqrt{1-\beta^2}} \frac{1}{c} \frac{dv}{dt} \\ \frac{1}{\sqrt{1-\beta^2}} \frac{d\beta}{dt} &= \frac{1}{\sqrt{1-\beta^2}} \frac{1}{c} \frac{dv}{dt} \\ \frac{1}{\sqrt{1-\beta^2}} \frac{d\beta}{dt} &= \frac{1}{\sqrt{1-\beta^2}} \frac{1}{c} \frac{dv}{dt} \end{aligned}$$

These relationships could be used to derive the relativistic

kinetic energy and the relativistic momentum.

The relativistic kinetic energy is given by

where m_0 is the rest mass, m is the relativistic mass, and

is the relativistic momentum.

Since the relativistic mass is given by

the relativistic momentum is given by

where γ is the Lorentz factor, v is the velocity, and c is the speed of light.

The relativistic kinetic energy is given by

where m_0 is the rest mass, m is the relativistic mass, and

is the relativistic momentum.

$$\frac{1}{\sqrt{1-\beta^2}} \frac{d\beta}{dt} = \frac{1}{\sqrt{1-\beta^2}} \frac{1}{c} \frac{dv}{dt}$$

where β is the relativistic velocity, v is the velocity, and c is the speed of light.

$$\frac{1}{\sqrt{1-\beta^2}} \frac{d\beta}{dt} = \frac{1}{\sqrt{1-\beta^2}} \frac{1}{c} \frac{dv}{dt}$$

and the relativistic momentum is given by

where m_0 is the rest mass, m is the relativistic mass, and

$$(6-1) \quad \frac{1}{\sqrt{1-\beta^2}} \frac{d\beta}{dt} = \frac{1}{\sqrt{1-\beta^2}} \frac{1}{c} \frac{dv}{dt}$$

where β is the relativistic velocity, v is the velocity, and c is the speed of light.

$$\frac{1}{\sqrt{1-\beta^2}} \frac{d\beta}{dt} = \frac{1}{\sqrt{1-\beta^2}} \frac{1}{c} \frac{dv}{dt}$$

by substitution of this value into equation 2-3 it is seen that the phase angle between input and output is exactly ϕ .

Now if two of these simple lattices are constructed with different values for ϕ the phase difference is $\psi = \phi_2 - \phi_1$. The expression for the tangent of the half angle becomes

$$\tan \psi/2 = \frac{Q \left[\left(\frac{rf_0}{f} - \frac{f}{rf_0} \right) - \left(\frac{f_0}{rf} - \frac{rf}{f_0} \right) \right]}{1 + Q^2 \left[\frac{rf_0}{f} - \frac{f}{rf_0} \right] \left[\frac{f_0}{rf} - \frac{rf}{f_0} \right]} \quad \begin{matrix} f_1 = f_0/r \\ f_2 = f_0 r \end{matrix} \quad (2-4)$$

At this point Luck ingeniously introduces four parameters

- 1) $f/f_0 = \tan 1/2 n$
- 2) $Q(r - \frac{1}{r}) = \tan 1/4 \psi_0$
- 3) $\frac{1}{Q^2} - (r - \frac{1}{r})^2 = 4 \cos(\sec^2 1/2 \sigma)$
- 4) $\tan 1/2 \sigma \operatorname{cosec} n = \tan 1/2 \theta$

By substitution of these values and reduction of the resulting equations the very simple equation for the behavior of the phase difference network is obtained.

$$\frac{\tan 1/2 \psi}{\tan 1/2 \psi_0} = \frac{\sin \theta}{\sin \sigma} \quad (2-5)$$

Using this expression the family of curves shown in Figure 11 is obtained which bear resemblance to the familiar universal resonance curves for a single tuned coupled circuit. By inspection of these curves it is obvious that for a specified maximum of phase deviation from a given constant phase difference, Q and r should be chosen to place the value of ψ_0 at somewhat less than the phase constant and n somewhat more than 90 degrees in order to achieve optimum bandwidth. The parameters used by Dome in his development corresponds identically with Q in its conventional

the following conditions are satisfied: (i) f is a function from X to Y ; (ii) f is continuous; (iii) f is injective; (iv) f is surjective; (v) f is a homeomorphism.

Let $f: X \rightarrow Y$ be a function. Then f is a function if and only if it satisfies the following conditions: (i) f is a function; (ii) f is continuous; (iii) f is injective; (iv) f is surjective; (v) f is a homeomorphism.

$$f(x) = \begin{cases} x & \text{if } x \in X \\ 0 & \text{if } x \notin X \end{cases}$$

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sense as used here.

Luck further reduced a number of similar lattice structures to the basic circuit used in this development and these are illustrated in the appendix (Appendix II). The fourth of these circuits is the one originally proposed by Dome. In terms of Luck's analysis the conditions required by Dome exist when

$$\psi_0 = 90^\circ$$

This network takes precedence over the others by virtue of the fact it can tolerate a load of resistive, capacitive, or inductive nature.

From experience with practical uses of this network, the following comments are made about the basic design (see Figure 8):

1) The tolerance of the two series arms are very critical; components used in these impedors should be of plus or minus 1%.

2) The value of the resistance in the shunt arm is also critical, but for some reason the capacitance is not. This is very fortunate since it permits feeding the output signal from the network directly into an amplifier stage. If a pentode amplifier is used the input capacitance may be quite large. In one particular instance the resistance R3 was 62.5 k and C3 was made variable from 480-575 mmf.

3) The equality of voltages in the phase splitter is not as critical as might be expected. The plate and cathode load resistors can be made plus or minus 5% tolerance without undue error being noticed.

The above observations are strictly of an experimental nature and no analytical explanation is offered.

The R-C feedback amplifier can also be used to obtain a precise 90 degree phase shift. While this device can be adjusted to give exactly

Figure 12
General Feedback Amplifier

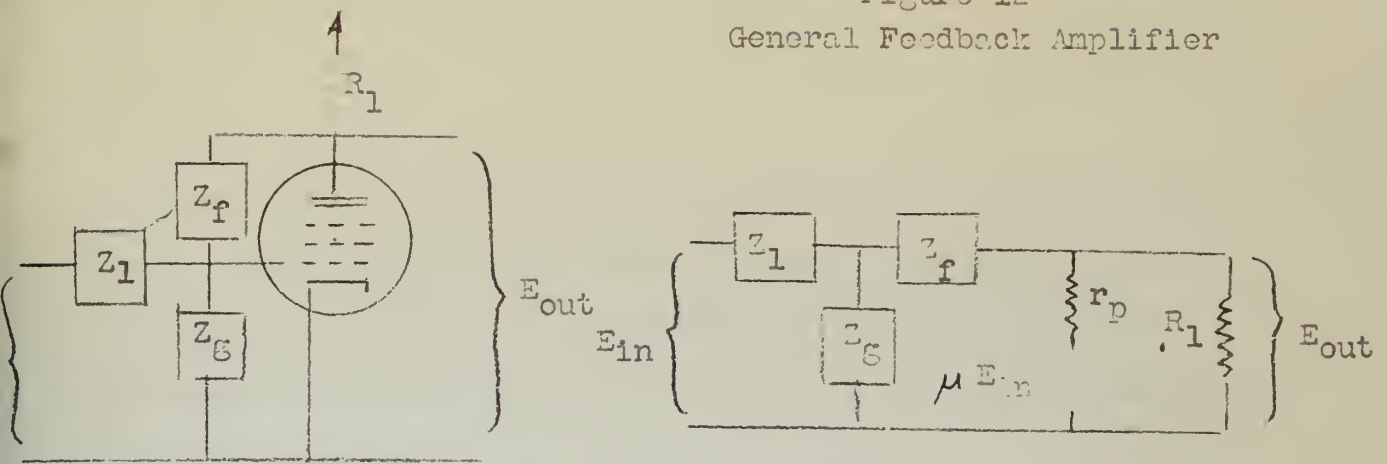
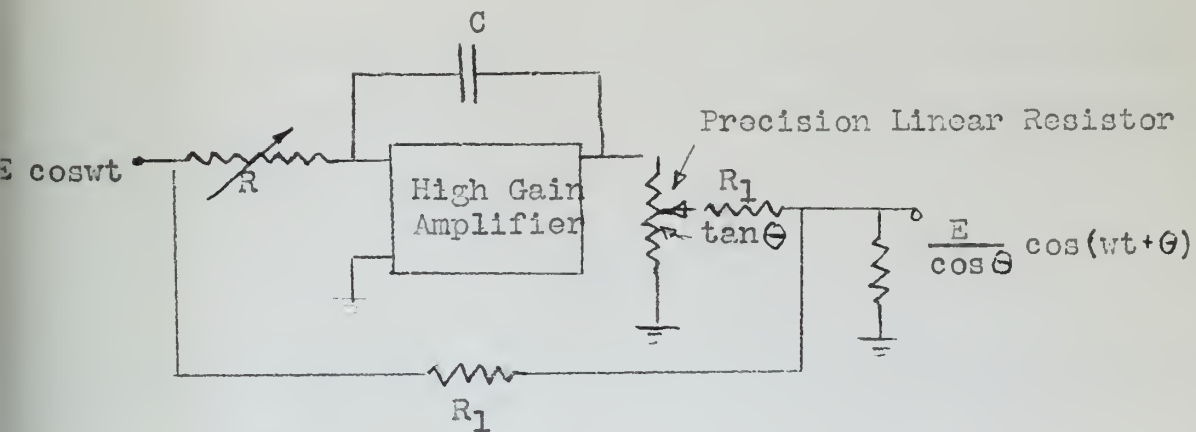
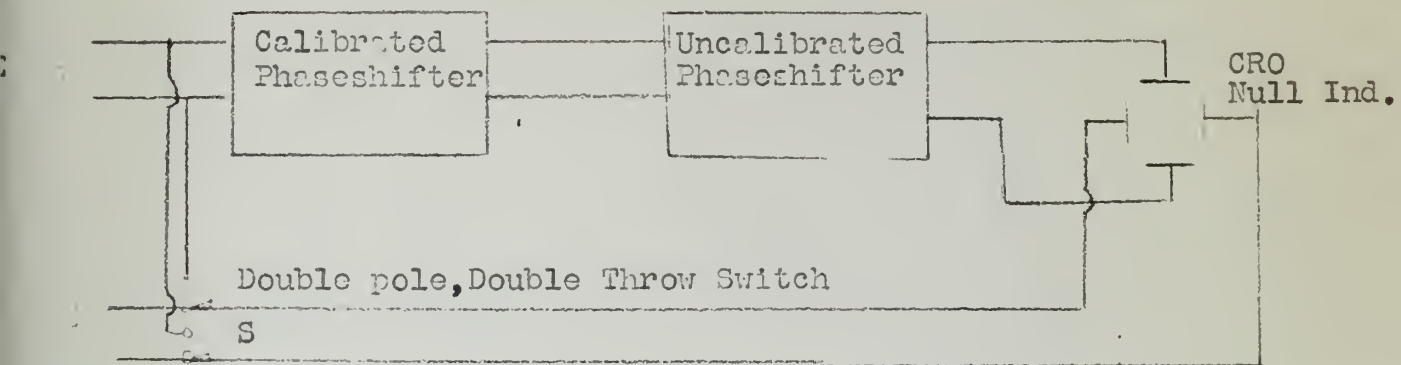


Figure 13
An R-C Feedback Amplifier Precision Phasemeter



90 degrees of shift at some particular frequency, it is very frequency sensitive and cannot be used when the signal to be shifted is complex. This system is used in a certain highly precise phasemeter which, in view of its purchasers, has been used almost exclusively in the field of radio navigation.¹⁰

The manner in which the shift is obtained is by adjusting the feedback network to provide exactly unity gain. With this condition the phase shift of the amplifier is 90 degrees. To verify this statement consider a single, high gain vacuum tube amplifier, Figure 12, arranged as a general feed back circuit. Following the analysis of Seely, page 155 op. cit, the complete expression for the gain of the circuit is

$$K_r = - \frac{Z_f}{Z_i} \frac{1}{1 - \frac{1}{K} - Z_f \frac{Z_i + Z_g}{Z_i Z_g} \cdot \frac{1}{K}} \quad (2-6)$$

and if Z_g can be taken as being very much greater than Z_i

$$|K_r| = \frac{Z_f}{Z_i} \times \frac{1}{1 - \frac{1}{K} - \frac{Z_f}{Z_i K}}$$

For a multistage amplifier very high voltage gains can be realized. The terms containing the factor $1/K$ can be assumed to vanish and the resultant expression for gain is

$$K_r = - \frac{Z_f}{Z_i}$$

When the gain is reduced to unity Z_f equals Z_i and the complex gain equation is

$$K_r = - \frac{1/j\omega C}{R} = - \frac{1}{j\omega CR} = \frac{1}{\omega RC} \angle 90^\circ$$

showing that a phase shift of exactly 90 degrees is obtained.

An interesting direct application of the R-C feedback amplifier in phase measurement was made by Raggazzini and Zadeh.¹⁷ A simplified diagram of their wideband audio phase meter is shown in Figure 13. The output of the phase shift amplifier is combined with the input voltage to give a resultant voltage which is directly related in phase to the mechanical angle of rotation of the precision resistance potentiometer R.

$$E_1 \cos \omega t + [E_1 \tan \theta] [\cos (\omega t + \pi/2)] = \frac{E_1}{\cos \theta} \cos (\omega t + \theta) \quad (2-7)$$

The application of this device is straightforward. With the calibrated phase shifter set at zero the uncalibrated phase shifter, which in this case is an R-C bridge, is adjusted to produce phase coincidence as indicated on the null indicating device. For null indication a CRO is used taking advantage of the expanded ellipse technique discussed in Appendix I. Switch S is then thrown to its alternate position and the calibrated phase shifter adjusted to regain the null condition. Since θ can be varied only ± 45 degrees by the precision potentiometer, additional ± 45 degrees of phase shift must be adjudged by noting the slant of the elliptical pattern.

This principle is utilized in the precision phase shifter referred to earlier with the following refinements:

- 1) A low level wattmeter is used to resolve ambiguity about 180 degrees giving the instrument an effective range from zero to 360 degrees phase shift.

- 2) A diode bridge is used as an indicating device instead of a CRO.

2. Wave Squaring and Limiting Circuits

Phase discriminators are normally amplitude sensitive, and it

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Figure 14

A Pentode Wave Squaring Circuit

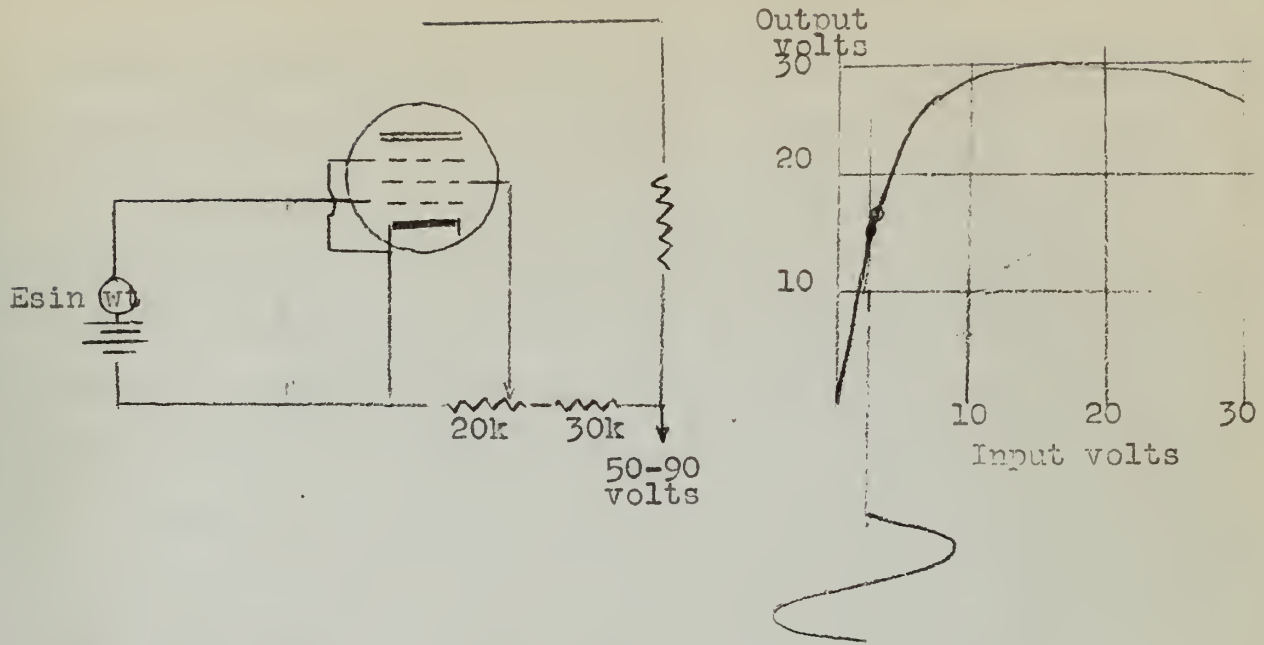
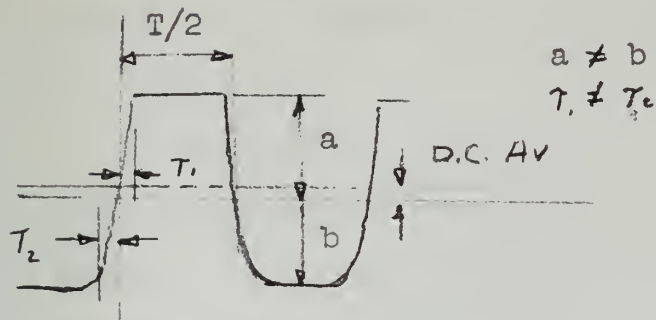


Figure 15

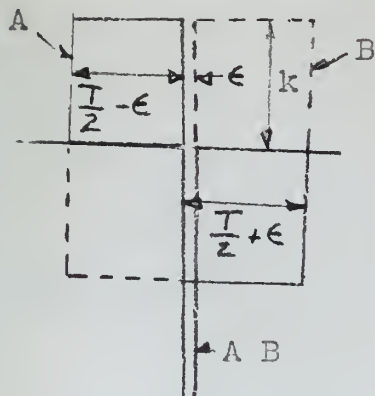


Due to improper location of quiescent point

Signal after further amplification and limiting showing that upon combination with another symmetrical signal 180 degrees out of phase a voltage

$$= \frac{\epsilon \times Zk}{T}$$

indicative of phase deviation from 180 degrees shift is produced.



Indicated Phase Error due to Dissymmetry in Wave Squaring

becomes necessary to remove amplitude variations in the reference and compared signal prior to discrimination. Possible circuits for signal limiting include diode clippers, multigrid tubes with either or both saturation and grid circuit limiting, pentode-diode combinations, synchronized multivibrators, etc. The most widely used limiter circuit has been a pentode amplifier operated in the region of saturation, a circuit generally associated with FM receivers, Figure 14.

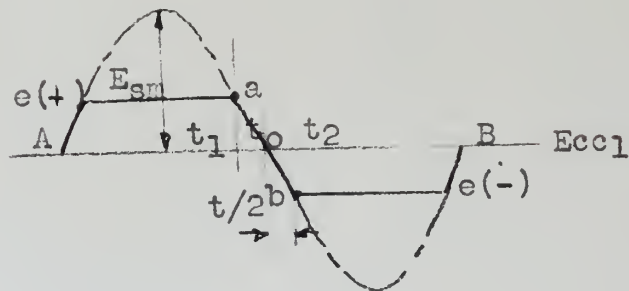
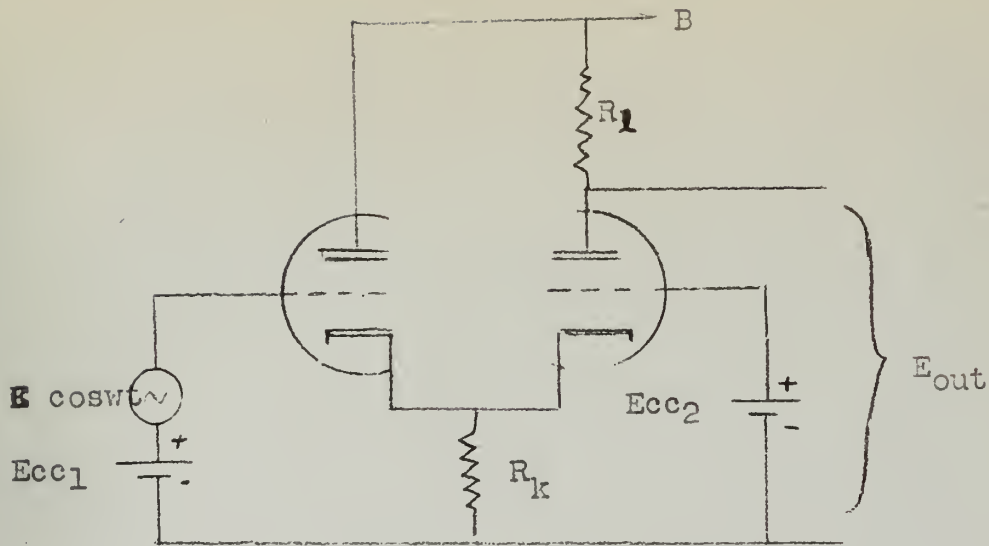
When identical limited waves must be produced in two channels operating conditions of the two limiter circuits must be very carefully controlled least dissymmetry be produced. This will result in an apparent phase difference out of the discriminator. Consider the condition illustrated in Figure 15; in this case a summing type of phase discriminator will produce a phase difference voltage directly proportional to the degree of dissymmetry.

A limiting circuit satisfactory up to several hundred kilocycles which is substantially independent of circuit and tube tolerances and changes is the cathode coupled clipper⁶, Figure 16. This very useful circuit should be considered anytime an audio limiting circuit is needed. Because of the importance of a satisfactory limiting circuit, not only in connection with phase measurement but as applied to speech handling equipment such as compressors and companders, this circuit is considered in detail in Appendix III.

Certain design considerations must be taken into account in the application of this circuit. When the signal levels obtainable are low

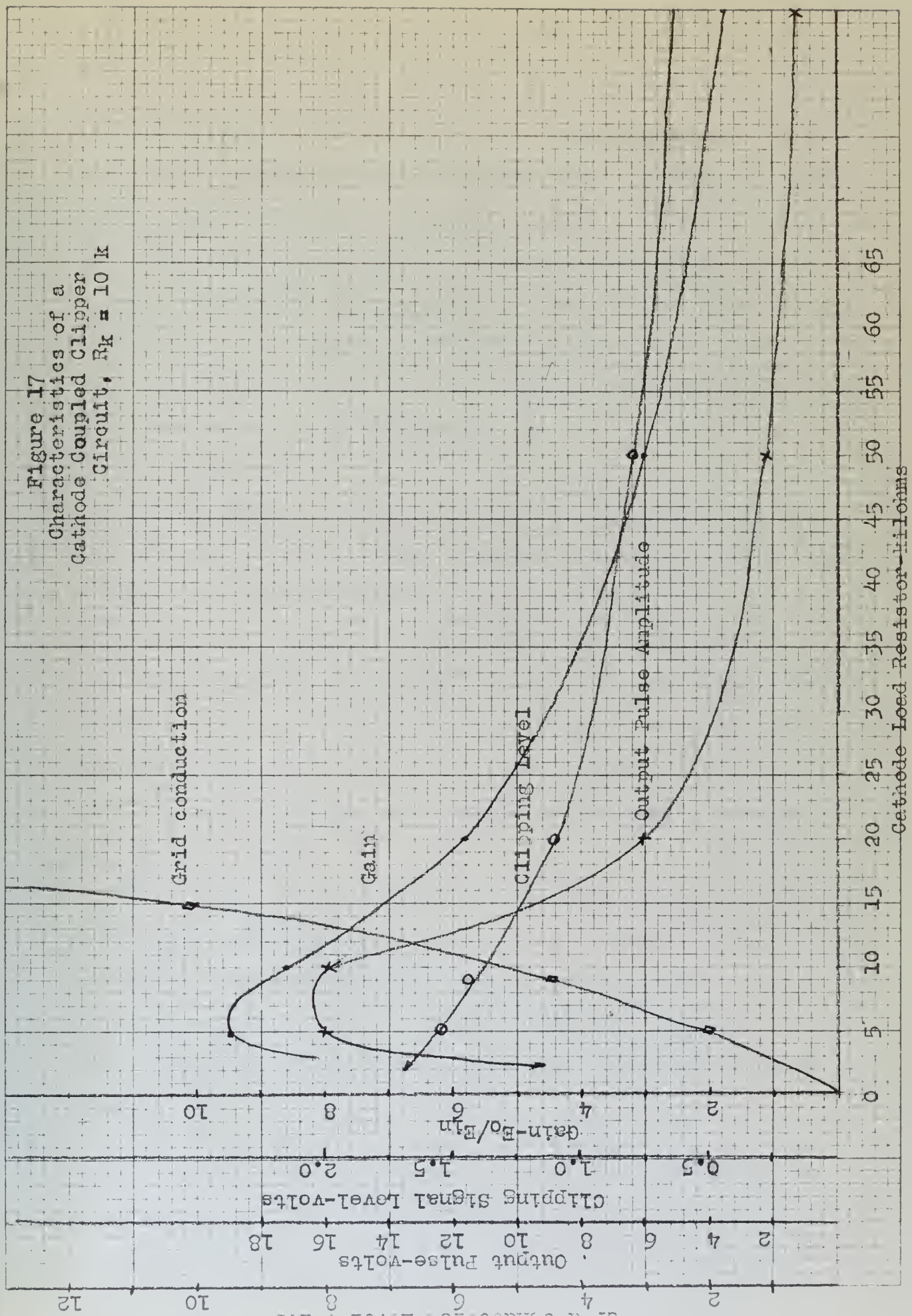
Figure 16

The Cathode Coupled Clipper Circuit



Clipping Action

Figure 17
 Characteristics of a
 Cathode Coupled Clipper
 Circuit, $R_k = 10\text{ k}$



it may be decided to sacrifice some of the clipping performance to obtain low level amplification in cascaded clipper stages. Clipping performance at high signal levels is not much affected by this. The value of the plate load resistor is a function of the upper signal frequency to be handled, and the output will be largely determined by the permissible clipper tube current. Figure 17 shows a typical set of performance curves for a clipper circuit using a 10k plate load resistor corresponding roughly to a upper frequency limit of 200 kcs.

3. Phase Discriminators

The discriminator is actually a specialized member of the large family of circuits which are phase selective. The discriminator owes its position of relative importance to the fact that it is able to tell both phase reversal and magnitude of actual phase shift. Discriminators can be classified generally as belonging to one of two groups 1) wave summing circuits and 2) phase sensitive rectifiers. Both of these types discriminate by adding the signals, the sum or difference of the signals to one of the basic difference between the two is the sense in which the addition is performed.

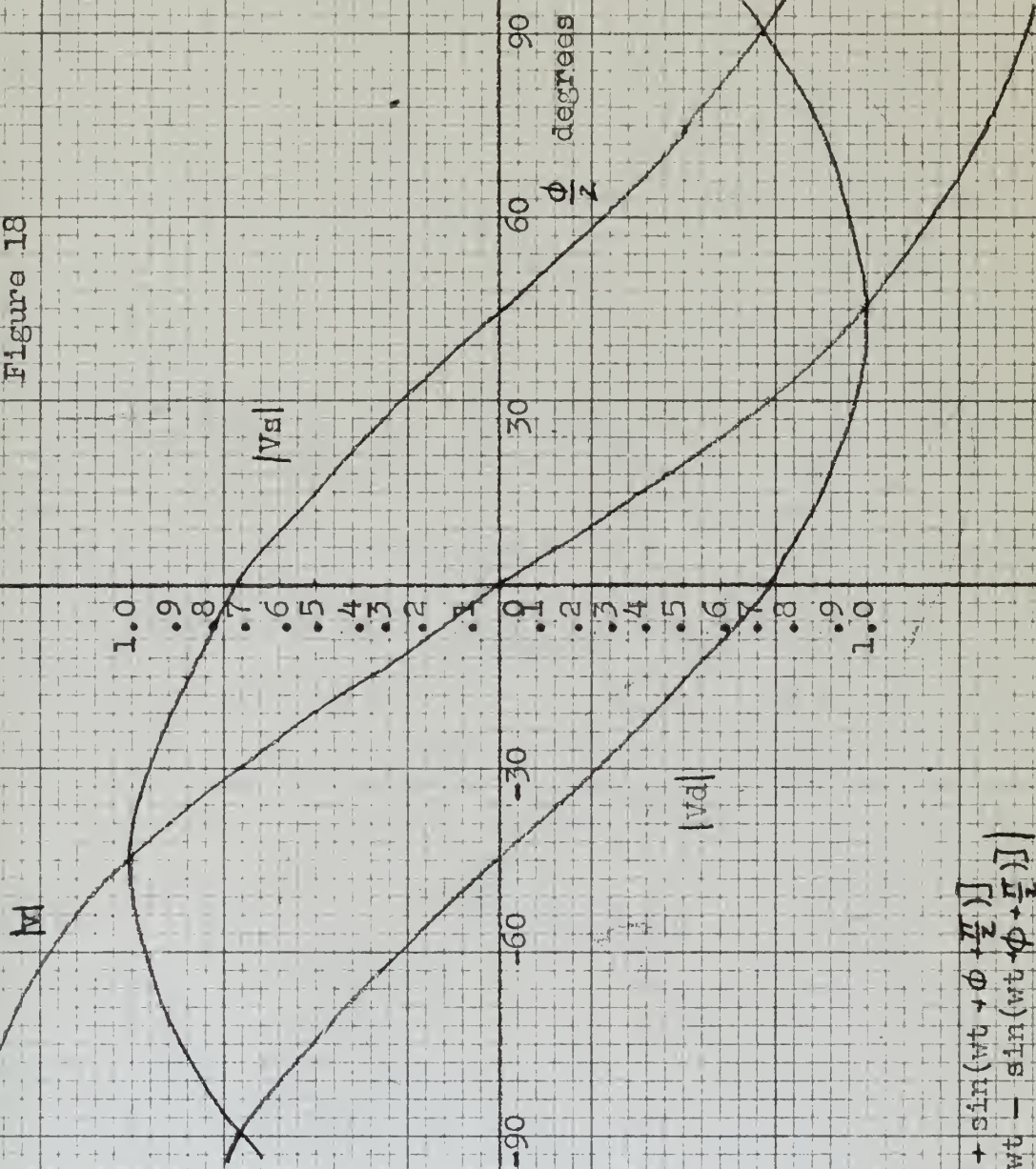
In the nature of an introduction to the subject assume that the two signals to be compared are sinusoidal and that the discrimination is to be performed by adding the sum of the signals to the difference. This is the method employed by the well known Foster-Seely discriminator circuit and the power frequency phase sensitive rectifier widely used in servo systems.

If

$$\begin{aligned}V_1 &= V_1 \sin(\omega t) \\V_2 &= V_2 \sin(\omega t + \phi)\end{aligned}$$

Phase Discrimination by Voltage Sum and Difference Addition

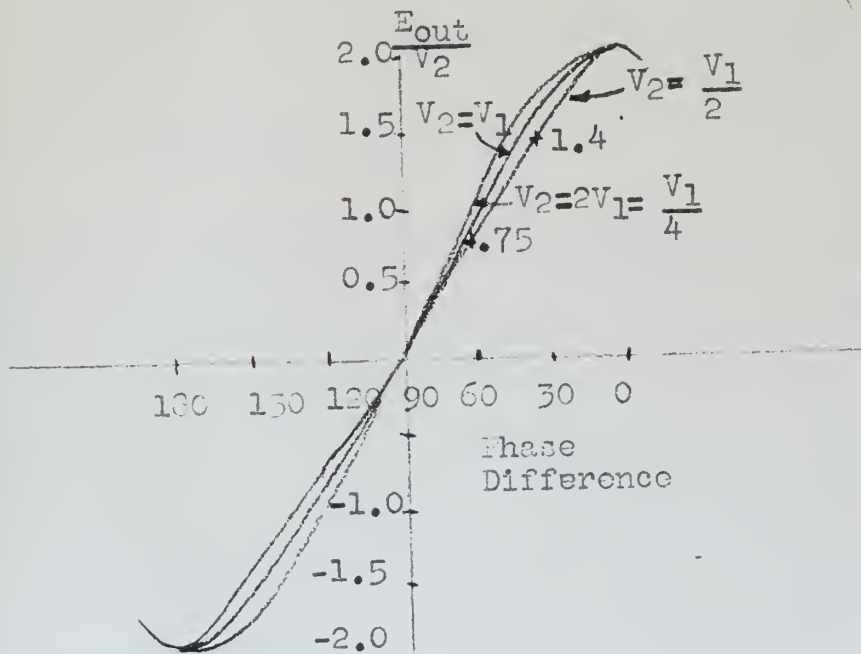
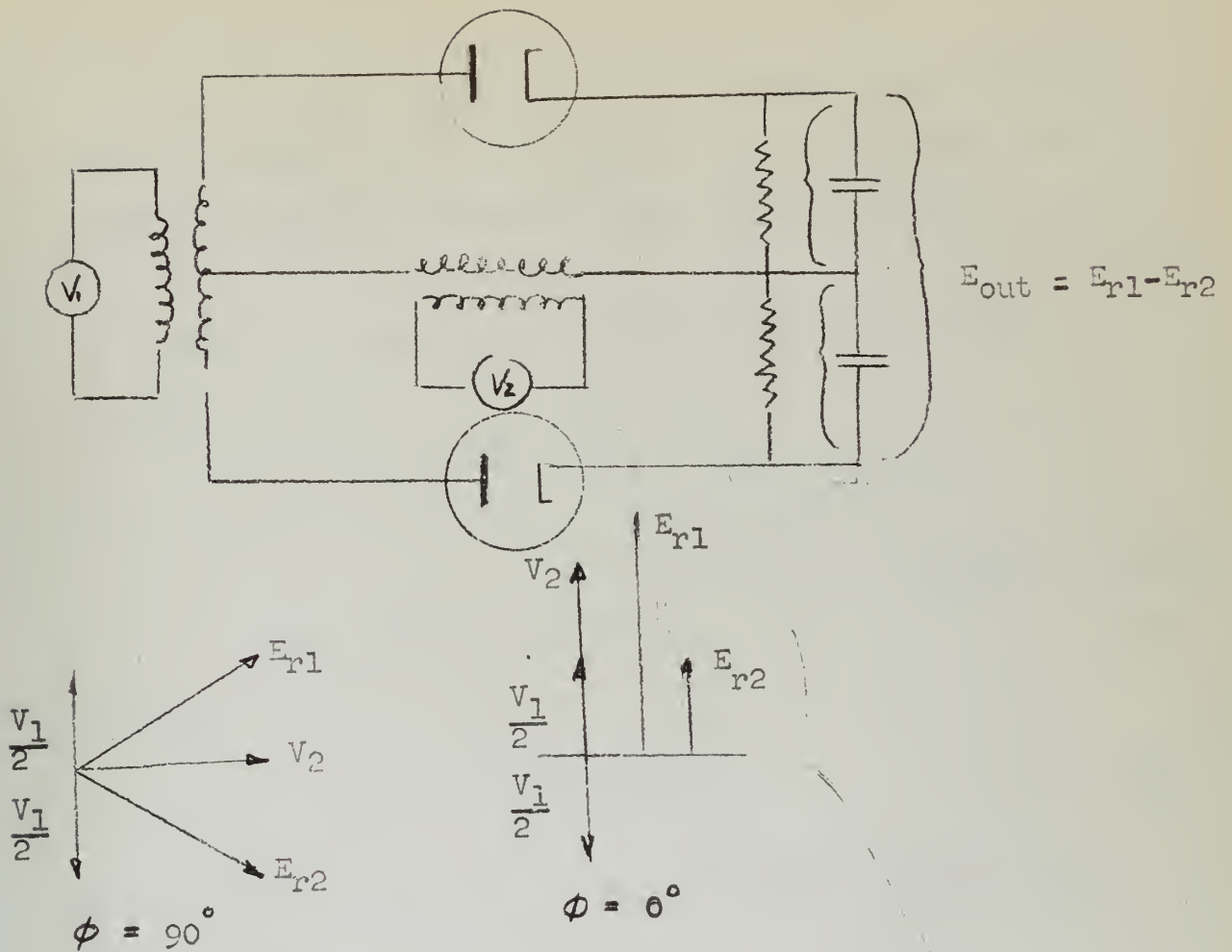
Figure 18



$$|V| = \left| V(\sin wt + \sin(wt + \phi + \frac{\pi}{2})) \right|$$

$$V[\sin wt - \sin(wt + \phi + \frac{\pi}{2})]$$

Figure 19
Output of a Peak Detecting Discriminator



Clearly the sum or difference of the two signals will vary as the amplitude of either one is varied, hence to render the output of the discriminator amplitude insensitive impose the condition that V_1 equal V_2 .

Adding the two signals and reducing by trigonometric means,

$$\begin{aligned}
 v_1 + v_2 &= V [\sin(\omega t) + \sin(\omega t + \phi)] \\
 &= V [\sin\{\omega t + \frac{\phi}{2} - \frac{\phi}{2}\} + \sin\{\omega t + \frac{\phi}{2} + \frac{\phi}{2}\}] \\
 &= V [\sin(\omega t + \frac{\phi}{2}) \cos \frac{\phi}{2} - \cos(\omega t + \frac{\phi}{2}) \sin \frac{\phi}{2} \\
 &\quad + \cos(\omega t + \frac{\phi}{2}) \sin \frac{\phi}{2} + \sin(\omega t + \frac{\phi}{2}) \cos \frac{\phi}{2}] \\
 &= 2V \cos \frac{\phi}{2} \sin(\omega t + \frac{\phi}{2})
 \end{aligned} \tag{2-8}$$

Similarly

$$v_1 - v_2 = -2V \sin \frac{\phi}{2} \cos(\omega t + \frac{\phi}{2}) \tag{2-9}$$

Now if a phase difference of 90 degrees is inserted in series with V_2 prior to addition and subtraction

$$v_s = v_1 + v_2 = 2V \cos(\frac{\phi}{2} + \frac{\pi}{4}) \sin(\omega t + \frac{\phi}{2} + \frac{\pi}{4}) \tag{2-10}$$

$$v_d = v_1 - v_2 = -2V \sin(\frac{\phi}{2} + \frac{\pi}{4}) \cos(\omega t + \frac{\phi}{2} + \frac{\pi}{4}) \tag{2-11}$$

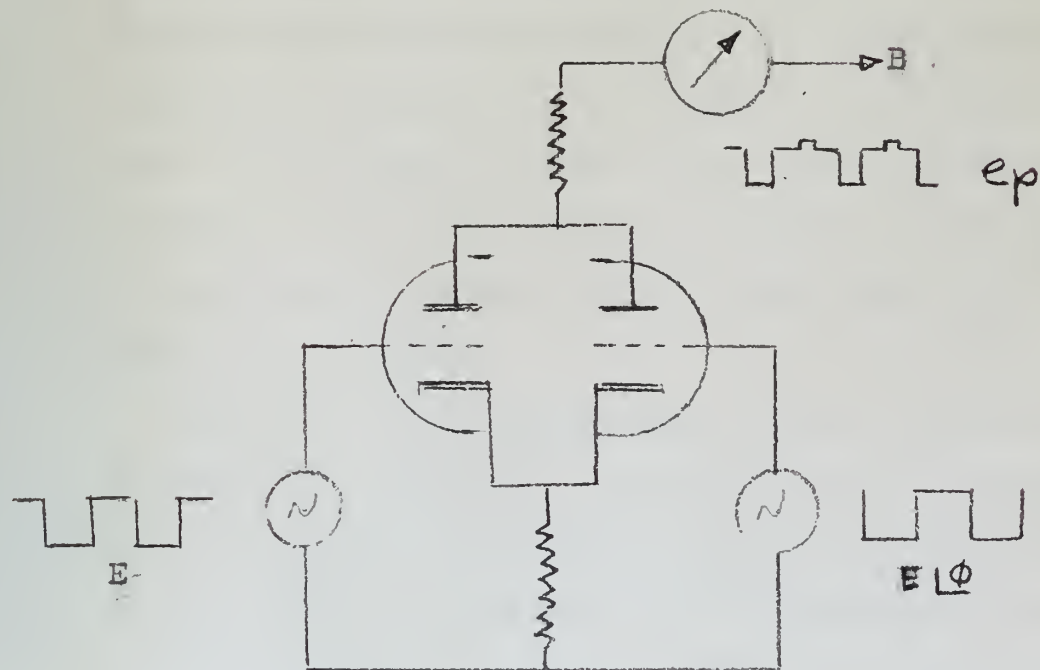
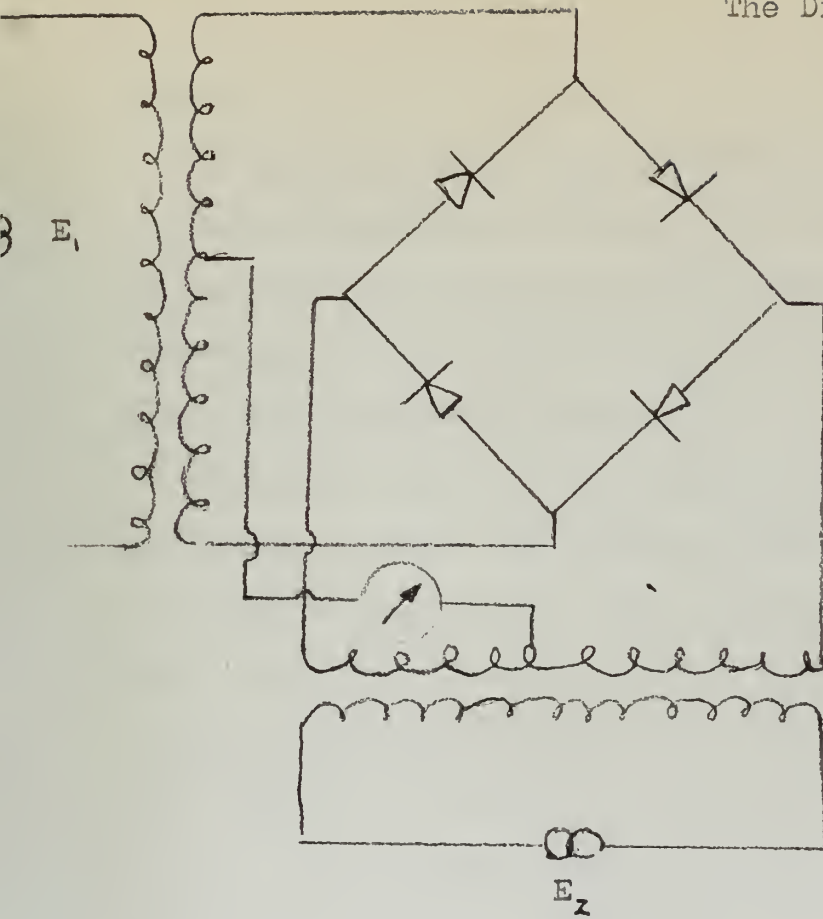
Discounting the time varying term and considering V_s plus V_d as a possible measure of the phase relationship, Figure 18, it is observed that V_s plus V_d is a sinusoidal function having its minimum value for ϕ equal zero. Identical results can be arrived at by assuming the 90 degree phase shift in series with V_1 . Since V_s plus V_d without the shift is a maximum for equal zero, the 90 degree shift provides for a function which has its maximum rate of change at the condition of phase coincidence, a desirable

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Figure 20

The Diode Bridge or Ring Modulator



A Vacuum Tube Circuit Having Properties Similar to the Diode Bridge

feature.

The well known Foster Seely discriminator is illustrated in Figure 19. The peak amplitudes of V_2 and $V_1/2$ are added and subtracted then the sum and differences detected and added together in the output of the circuit. If V_2 is not equal to V_1 this output varies from a sinusoidal and approaches linearity for V_2 equal $V_1/2$.

The diode bridge, or ring modulator, is probably the most sensitive discriminator circuit. Objectionable features are that it must be transformer fed and that the diodes must fairly well matched. When the phase of one signal is first shifted 90 degrees this type of discriminator can give an unambiguous output from plus or minus 90 degrees of phase shift, Figure 20. To verify this statement consider that the indicating device is a center scale, average reading milliammeter, and that the signal sources are in phase. By tracing out the current flow it is seen that its direction through the meter is always in one direction only giving maximum deflection in one direction. At 90 degrees of phase difference the average current through the meter is zero, and at 180 degrees the current is maximum in the reverse direction. Furthermore, if the signal waveforms are square and of equal amplitude the average current is a linear function of the phase difference.

In the use of this circuit damping resistors across the transformer secondaries are a necessity. They should be of small tolerance. Trouble may also be encountered in dissimilarity of the diodes in which case precision resistors across the diodes to degenerate the discrepancies in back resistance may prove helpful. Useful information as to the details

of diode selection and procedure in system application of this circuit is to be found in literature on speech equipment used for telephone and single sideband radio transmission.

The transformer required is an expensive item since it must necessarily have a fairly (minus 3 db.) flat response. If, for example, the signal sources were square waves of frequency from 30 to 10,000 cycles the transformer would have to be flat from 3 cycles to 100 kcs.

For direct addition of signal voltages the basic vacuum tube circuit is a common plate or common cathode summing chain. To a first approximation which is sufficiently accurate for most engineering applications

$$\text{(Plate)} \quad \frac{1}{R_e} \sum_n e_n = - \frac{e(\text{sum})}{K} \frac{n+1}{R_e} - \frac{e(\text{sum})}{R_e} ; e_{\text{sum}} \doteq - \sum_n e_n \quad (2-12)$$

and

$$\text{(Cathode)} \quad e(\text{sum}) = \frac{\mu Y_p}{n(\mu+1)Y_p + Y_K} \sum_n e_n ; e(\text{sum}) \doteq \frac{1}{n} \sum_n e_n \quad (2-13)$$

When two sinusoidal voltages of the same frequency are added the resultant is another sinusoidal of the same frequency and of amplitude determined by the relative amplitudes and phases of the two signal voltages. If the amplitudes can be held invariant, the output sum voltage becomes a measure of the relative phase. A phasemeter employing this principle of discrimination will, in the absence of any other operations performed, have an output which is sinusoidal and of zero magnitude when the phase difference is 180 degrees. Maximum output and minimum sensitivity to phase difference occurs at phase coincidence which is considered to be undesirable since, a priori, it would seem the most important observation to be made is of

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$$(2) \quad \frac{1}{n} \sum_{i=1}^n \frac{1}{x_i} = \frac{1}{n} \sum_{i=1}^n \frac{1}{x_i} \quad (3)$$

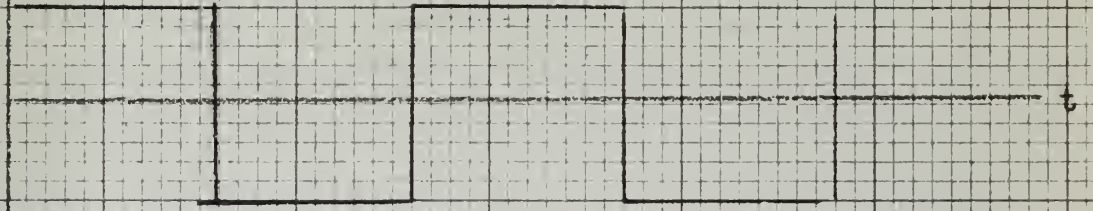
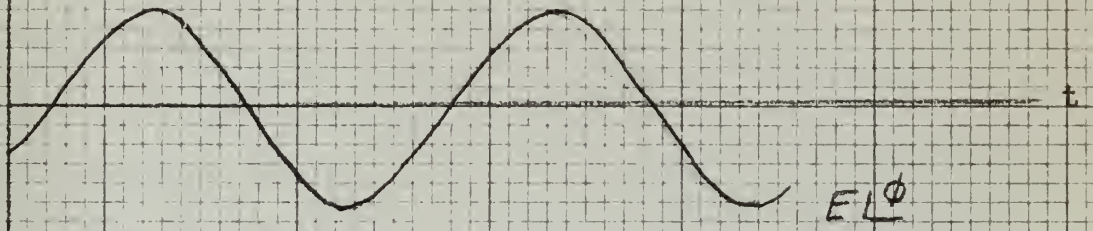
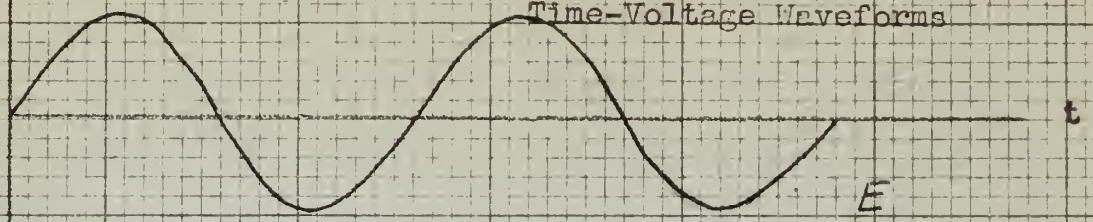
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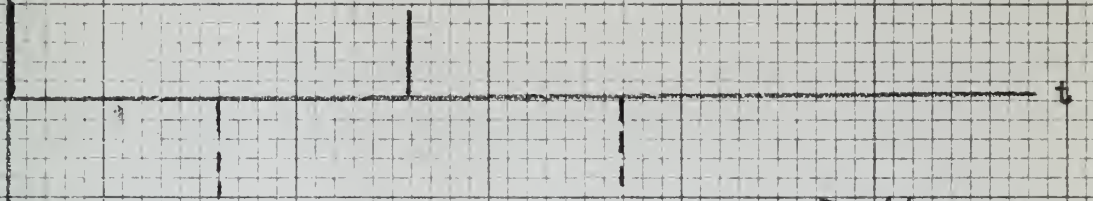
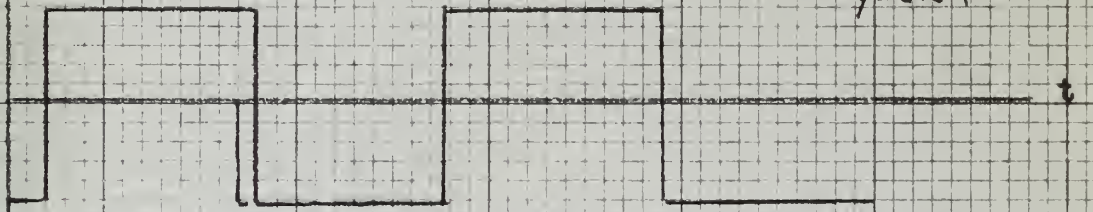
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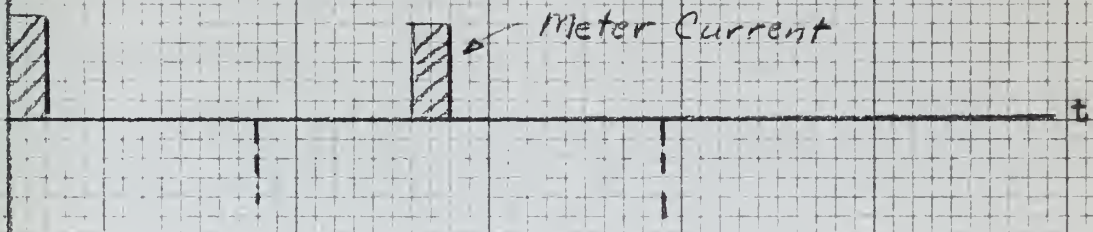
Figure 21
Technology Institute Phasemeter
Principle Illustrated by
Time-Voltage Waveforms



Squared



Differentiated



the similarity of two characteristics rather than the dissimilarity.

Florman's phase meter⁵ employs this method together with the basic principle of the Technology Institute Phasemeter to resolve ambiguity about 90 and 270 degrees. This last instrument is possibly the only truly portable instrument in anything resembling common use today and employs a unique principle worthy of some discussion.

The Technology Institute Phasemeter⁹ can be thought of as a summing device in that it adds the on-time of a bistable circuit to the off-time. The reference and measured signals are run through a series of amplifiers and cathode coupled clipper circuits to provide an essentially square wave. These signals are differentiated and the resulting voltage pulses used to trigger an Eccles-Jordan circuit from one state to the other. The indicating device is an average reading milliammeter inserted in the cathode circuit of one of the tubes in the Eccles-Jordan circuit. Provision must be made to insure that the bi-stable circuit will return to one particular condition in the absence of signals. This principle is illustrated in Figure 21.

The phasemeter of Krause and Watson⁸ gets around the 180 degree null condition by use of a cathode follower in one channel to insert an additional 180 degree shift. The effect on the summing amplifier of amplitude changes due to insertion of this cathode follower is open to conjecture.

The very useful circuit of Figure²⁰ can provide an output which is a linear function of phase difference of the input signals when these signals are square waves. This makes the circuit possess identical performance characteristics as the diode bridge for similar signal

Figure 22

Some Typical Summing Amplifiers

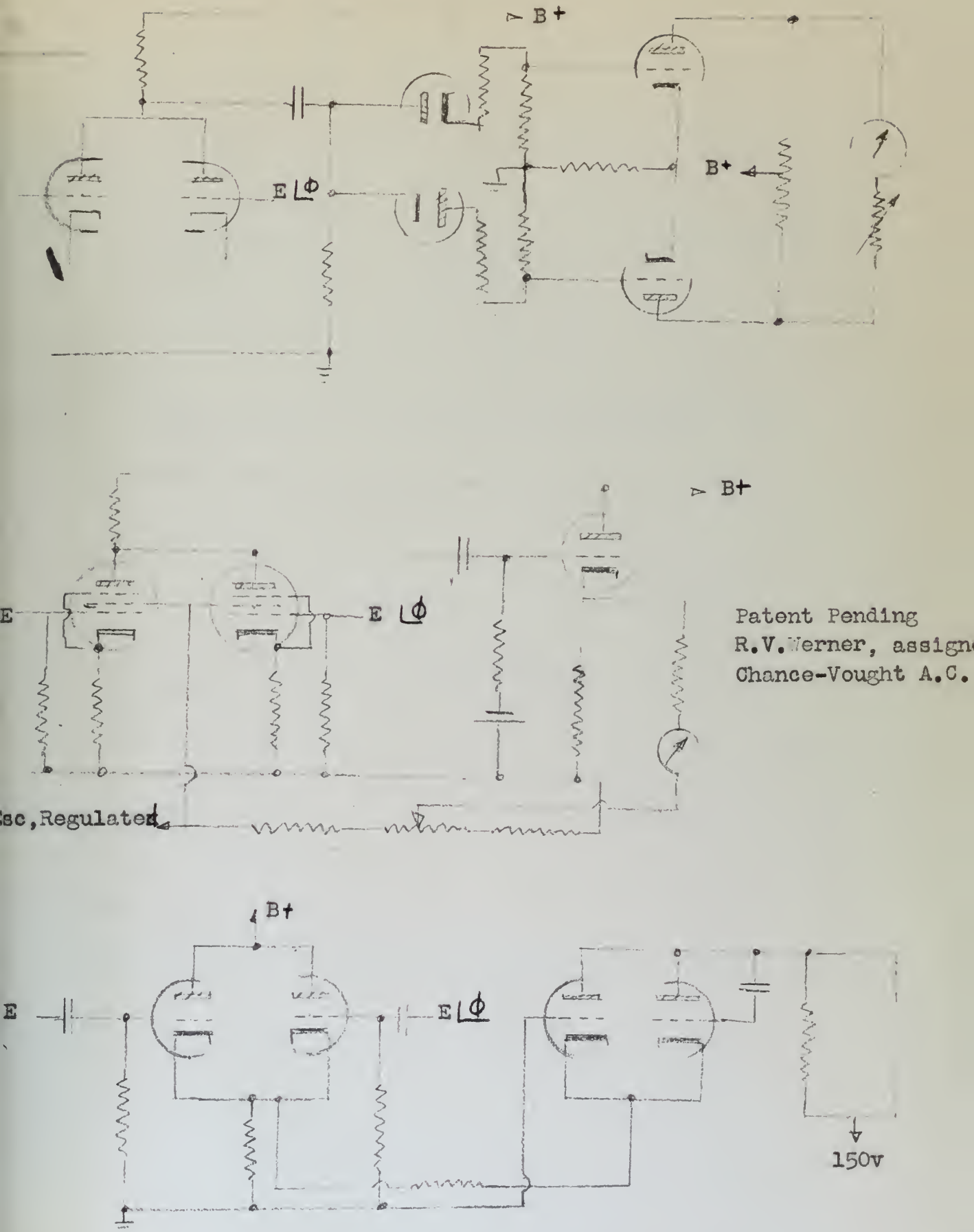
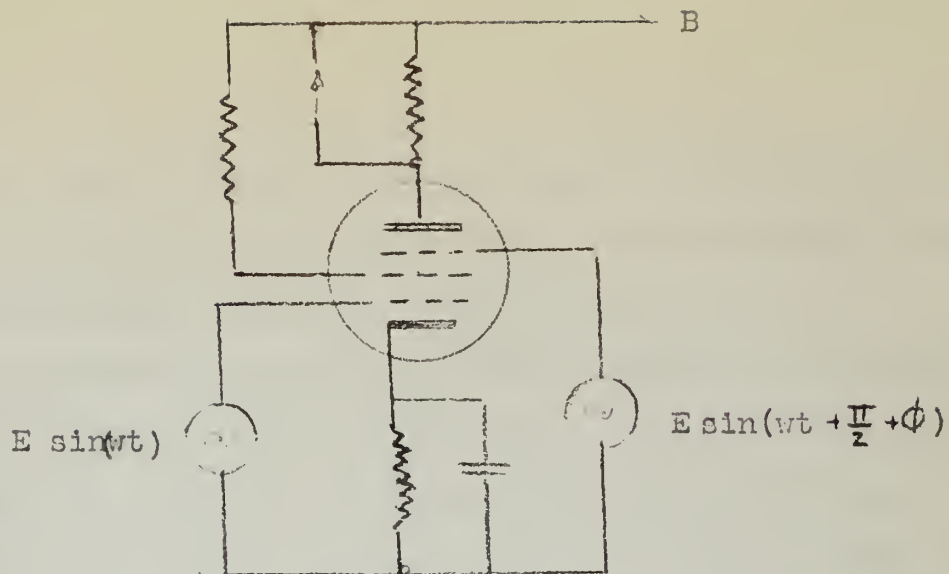
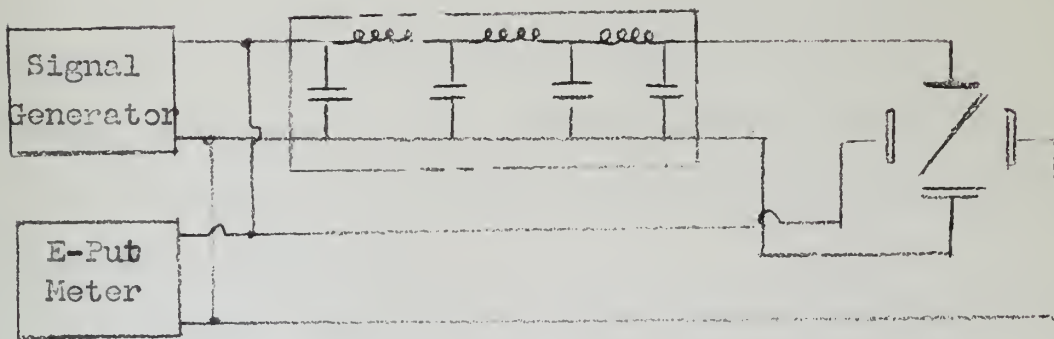


Figure 23



A Phase Sensitive Circuit Using a Dual Control Tube

Figure 24



Method of Obtaining the Base Frequency for a Standard Lag Line

sources. This is accomplished by choosing the cathode resistor to be of such magnitude as to degenerate the gain of each section to one half value when signals are applied to both grids.

A large number of practical summing amplifiers are currently in use. Several particular circuit configurations are shown in Figure 22 for purposes of illustration. In each case the essential difference is only in the metering circuit, a basic common plate or common cathode summing chain being used for addition of signal voltages.

The first circuit employs full wave rectification and a push-pull triode amplifier to isolate the meter circuit from the summing circuit and amplify the output of the summing amplifier. The second circuit uses a cathode follower in the conventional manner to isolate the metering circuit. In the third circuit the metering cathode follower is normally cut off. With the application of signal voltages decrease in current through the cathode summing resistor permits current to flow in pulses through the metering circuit. Positive feedback in the second section of the metering tube insures rapid conformance of the metering circuit to changes in signal phase.

If the signal amplitudes are large enough to cut off a dual control tube such as a 6AS6 or 6BN6 the tube can be used as a summing amplifier type discriminator directly. A milliammeter in the plate circuit or voltmeter across the plate load serves as an indicator, Figure 23. For large grid signals these tubes are saturated and cut off rapidly by the signals at either grid and the plate wave form is essentially that shown for Figure 20. The big problem is to feed large signals into the grids of these tubes without getting into trouble with grid current. If the

signals are fed in through an R-C coupling circuit signal bias is built up. Cathode follower or transformer coupling is to be preferred. Also this trouble could be circumvented if the grid signals were square in from at the onset. In this manner the tube could be rapidly cut on and off without grid current flowing.

CHAPTER IV

STANDARDS FOR PHASE MEASUREMENT AND CALIBRATION

No primary standard for phase measurement is available. That is it is not possible to relate the property to any of the canonical quantities directly such as current measurement by Faraday's electrolysis method or voltage measurement by standard Weston cell. This would seem to follow from the fact that while it is possible to note the passage of time in a very accurate manner it is not possible to advance or retard time by an arbitrary amount.

Two highly precise secondary standards will be discussed. They are 1) an artificial transmission line which can be accurately calibrated at one or more frequencies for use at any frequency within the limits of its construction and 2) a phase shifter of the goniometer type which can be made highly accurate for one given frequency. It is felt that these two could satisfy any need for a phase standard that might arise.

1. A Lag-line Standard.

A simple lumped constant multi-section transmission line can be built to any electrical length desired. As an arbitrary criterion say the line is to be 360 electrical degrees long at the lowest frequency to be encountered. The line is a multisection, constant-k filter which obeys the well known laws for a network of this type.

$$\tau = \sqrt{LC} \quad ; \quad Z_0 = \sqrt{\frac{L}{C}} \quad ; \quad f(\text{cutoff}) = \frac{1}{\pi \tau}$$

A jig and CRO arrangement described in Appendix I can be used to assure that the sections are identical.

CHAPTER IV

THEORY OF THE ELECTRIC CIRCUIT

The circuit is defined as a closed path in which the electric current flows. It is one of the most important concepts in the theory of the electric circuit. The circuit is defined as a closed path in which the electric current flows. It is one of the most important concepts in the theory of the electric circuit. The circuit is defined as a closed path in which the electric current flows. It is one of the most important concepts in the theory of the electric circuit.

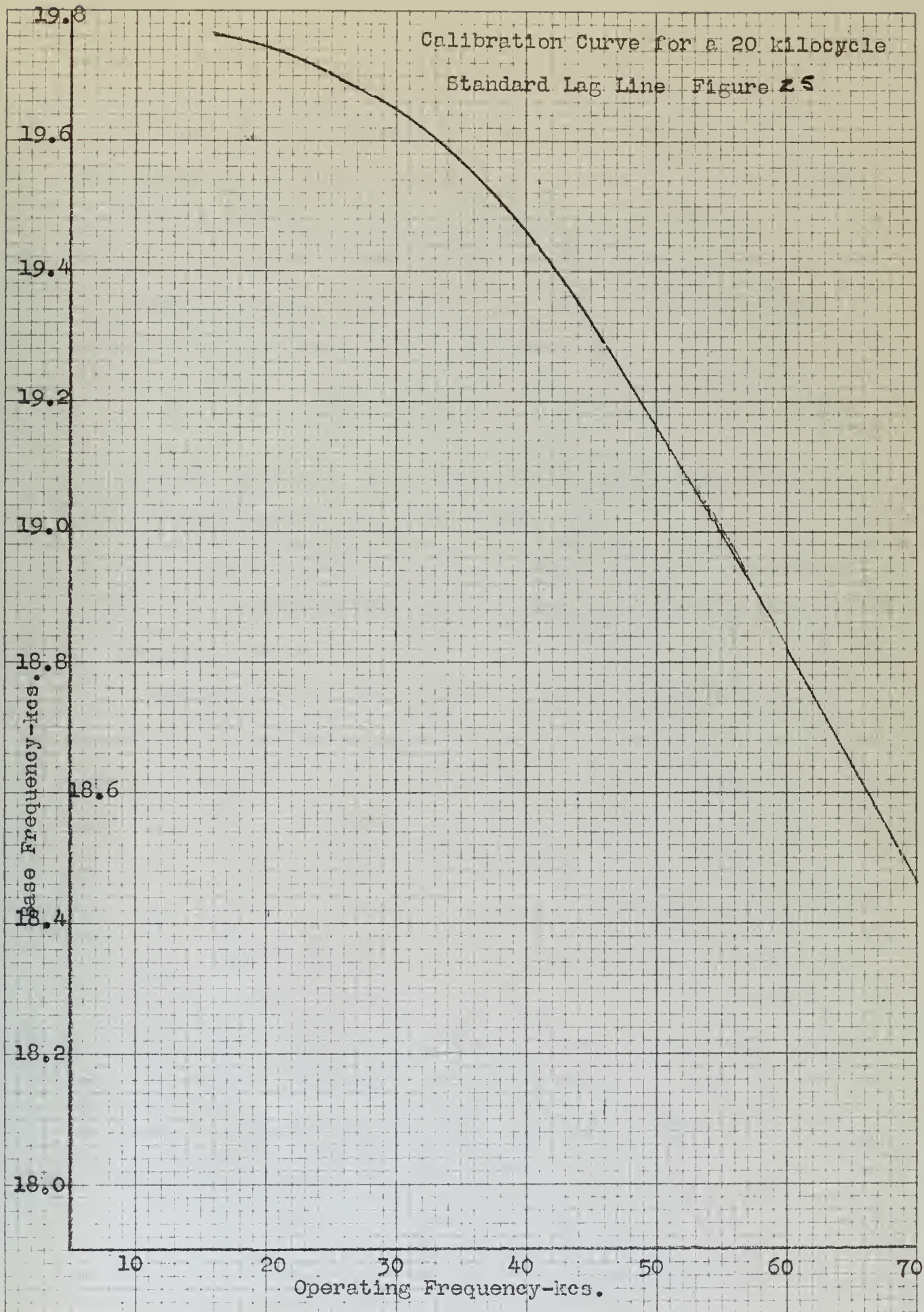
The circuit is defined as a closed path in which the electric current flows. It is one of the most important concepts in the theory of the electric circuit. The circuit is defined as a closed path in which the electric current flows. It is one of the most important concepts in the theory of the electric circuit. The circuit is defined as a closed path in which the electric current flows. It is one of the most important concepts in the theory of the electric circuit.

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$$I = \frac{V}{R}$$

The circuit is defined as a closed path in which the electric current flows. It is one of the most important concepts in the theory of the electric circuit. The circuit is defined as a closed path in which the electric current flows. It is one of the most important concepts in the theory of the electric circuit.

Calibration Curve for a 20 kilocycle
Standard Lag Line Figure 25



Once the line has been constructed it is only necessary to calibrate its electrical length by use of a CRO and signal generator. (Figure 24) A calibration chart is then constructed for use in the frequency range using as one argument a "base" frequency at which the electrical length is an integral multiple of π radians. The phase lag is then simply related to the base frequency by the equation

$$\theta = \frac{f}{f_0} \theta_0 \quad (\theta_0 \text{ is electrical length at } f_0) \quad (3-1)$$

A typical curve for a 20 kcs standard line having an electrical length of 360 degrees is shown in Figure 25.

The recommended construction of this line is with use of inductances of litz wire over powdered iron torroidal cores and silver mica condensers.¹⁹

2. A Goniometer Phase Standard

An extremely accurate phase standard of the goniometer type can be constructed. This method of phase control is well known and one of its most important uses which comes to mind is in the Meacham range unit widely used in radar. However, the goniometer capacitor used in the Meacham unit is expensive and since the principle can be adapted to use with either resistance or inductive elements a resistance goniometer²¹ would seem to be more attractive for most engineering uses, Figure 26.

There is an inherent error in this device due to the fact that there is a circulating current in the resistance elements of a round potentiometer and an additional current in the slider circuit. If the slider current can be neglected the phase angle error can be obtained by graphical solution of the equation

Once the line has been constructed it is only necessary to call-

lines the standard length to one of a 100 and 1000 standard. (Figure 14)

A standard length is then constructed for use in the standard lines
 being used to construct a line, standard of which the standard length
 is an integral multiple of 10 standard. The lines are in this manner
 related to the line segment in the standard.

$$(I-1) \quad \frac{L}{L_0} = \frac{L_0}{L_0} \quad \frac{L}{L_0} = \frac{L_0}{L_0}$$

A typical curve for $L = 10$ and standard line being an elliptical

length of the curve is shown in Figure 15.

The standard construction of this line is with use of standard
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 A. A standard line standard

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Resistance Goniometer and Typical
Quadrature Voltage Generator Circuit
Figure 26

B+

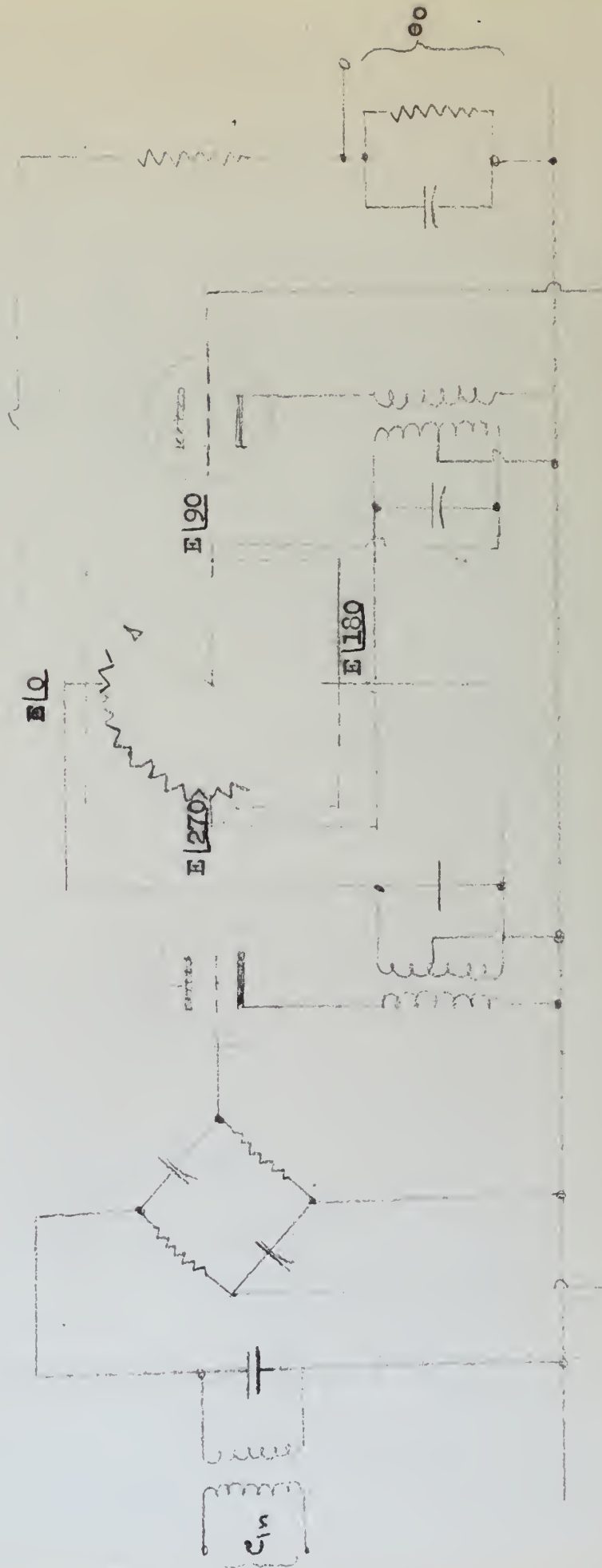


Figure 26

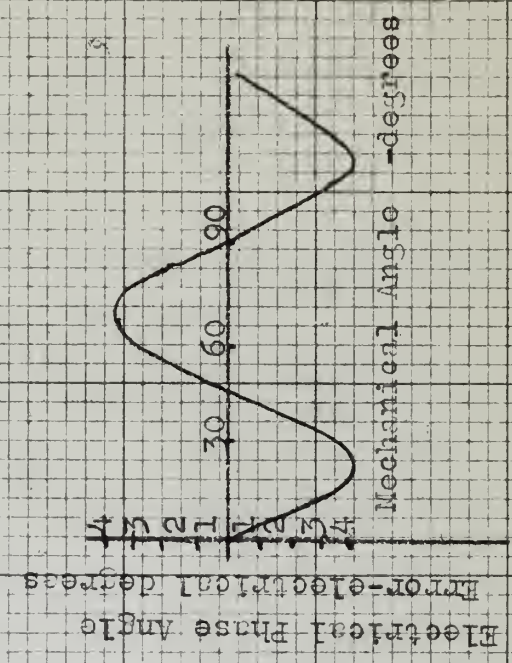
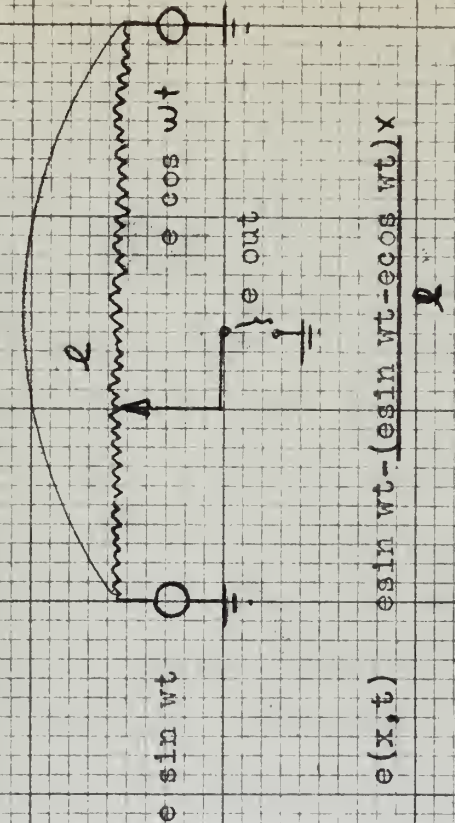
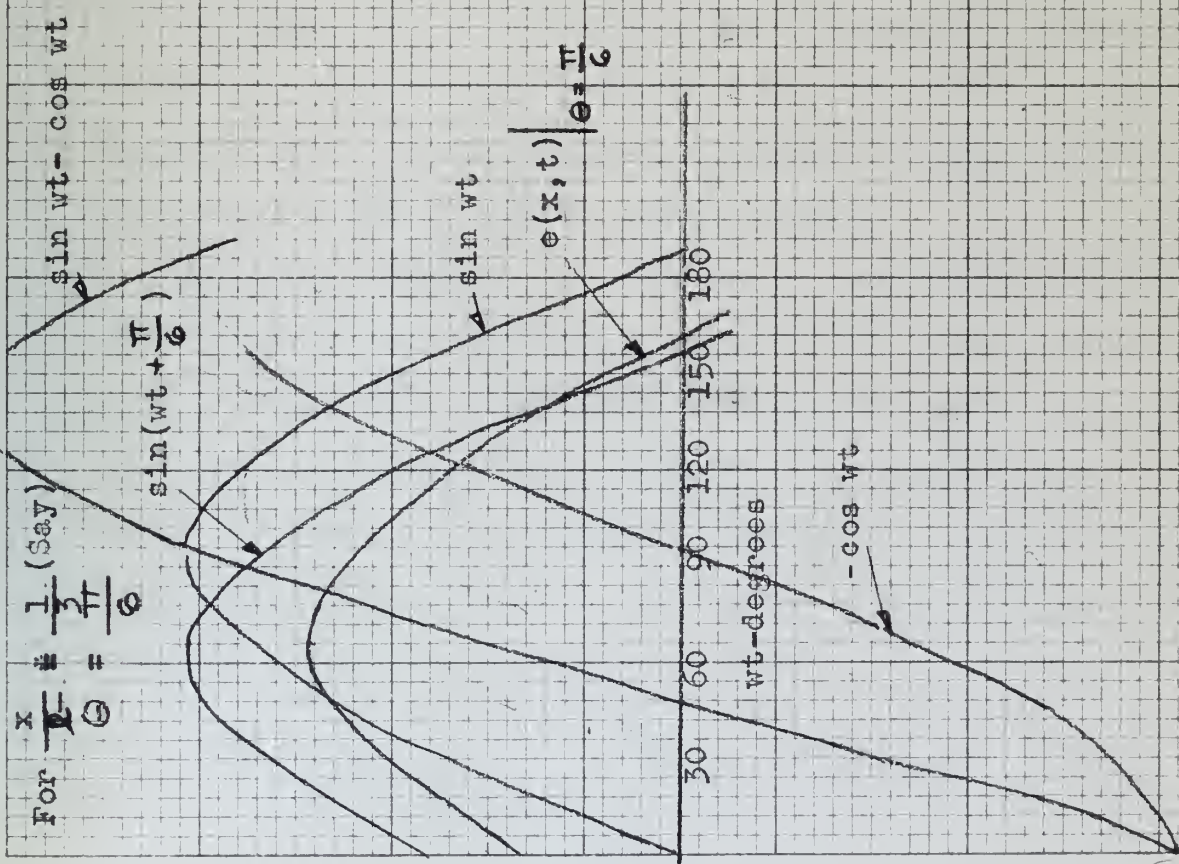


Figure 27
Resistance Coniometer
Output Phase Error

$$x = \frac{\theta (2\ell)}{\pi}$$

$$\xi(x) = E \sin\left(\omega t + \frac{\pi x}{2\ell}\right) - E \left[\sin(\omega t) - \frac{x}{\ell} (\sin(\omega t) - \cos(\omega t)) \right] \quad (3-2)$$

where x is the distance along the potentiometer from a tap and ℓ is the distance between two taps. A curve of phase error versus mechanical angle θ is shown in Figure 27. This error can be reduced to a very small amount by appropriate frequency division.

It may be shown that if the potentiometer is made square and if the slider still moves in an arc the error becomes zero.

The idea of a resistance goniometer is encouraged by the fact that there are commercially available very accurate multiturn, linear potentiometer (Helipot). Unfortunately most of these potentiometers have characteristics resembling transmission lines at frequencies above power frequencies and the resistance looking back from the taps should be as high as possible to avoid reflections and discontinuities on this equivalent line. No theoretical analysis is possible because the exact equivalent circuit of the multiturn potentiometer is not known to a sufficiently accurate degree at high frequencies.

Inductance goniometers are also used in a number of applications, notably in radio direction finding equipment. The general development of their circuit follows the same line of reasoning as the resistance goniometer and the capacitance goniometer to be discussed later. If a choice is possible, the capacitance goniometer is to be preferred since the coupling coefficients of the coils cannot be maintained with the tolerances possible in the capacitance device.

Capacitance Goniometer and Equivalent Circuit

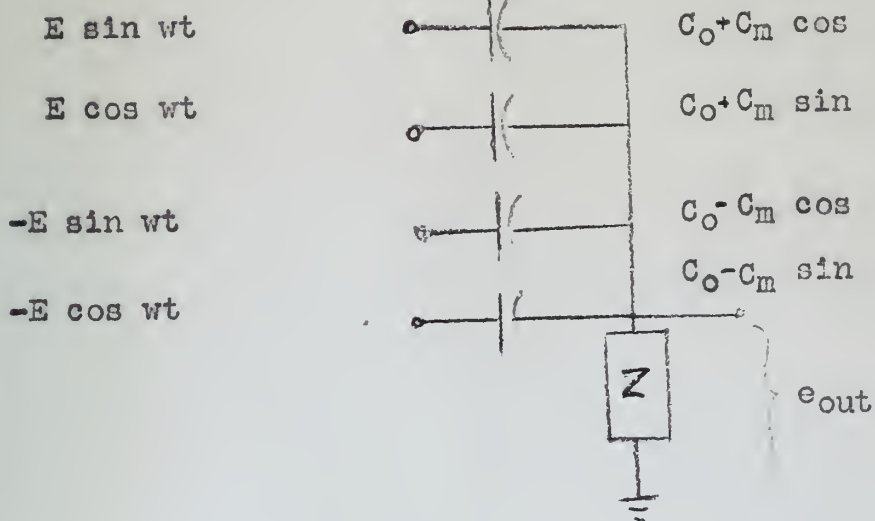
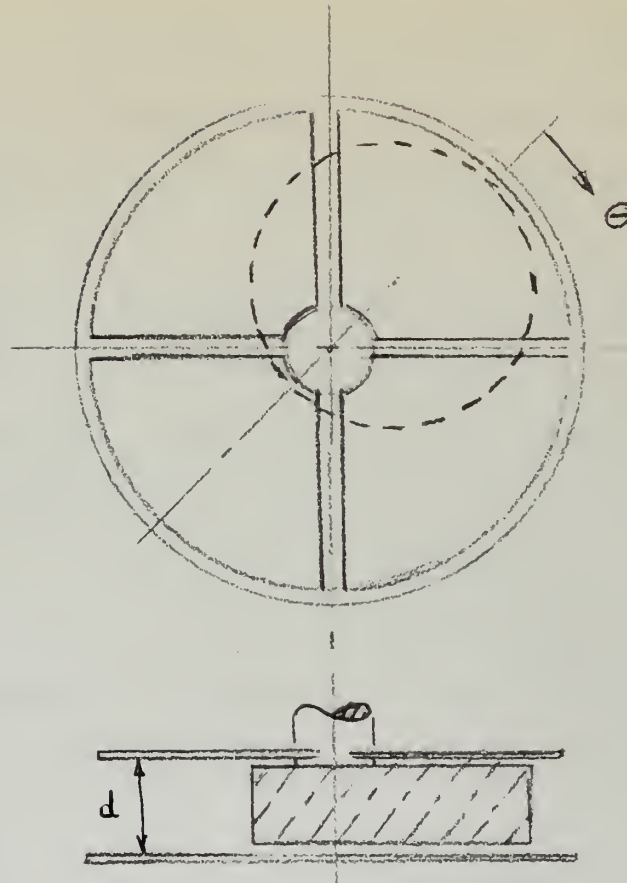


Figure 28

The capacitance goniometer has had notable success in radar and omnirange beacon circuits, however, the precision capacitor is an expensive item. In the figure 28 the capacitance of the individual sections is

$$C = \frac{kA}{d} = \frac{k(A_{ir}) \times Area(A_{ir})}{d} + \frac{k(Diel.) \times A(Diel.)}{d} \quad (3-3)$$

The dielectric constant is made to vary sinusoidally with the angle of mechanical rotation θ , that is

$$C(\text{Section}) = C_0 + C_m \sin \theta$$

Then by writing the sums of the branch currents as the current through the load impedance and solving for the output voltage, it is seen that

$$E(\text{out}) = \frac{2Em}{X(C_m)} \sin(\omega t + \theta) + \text{D.C. Term.} \quad (3-4)$$

The electrical phase angle of the output signal has been made to vary directly as the mechanical angle.

The objective function is not strictly convex in the parameters.

However, the function is strictly convex in the parameters of the linear model. In this case the minimization of the function is

linear.

$$(5-1) \quad \frac{\partial L(\theta)}{\partial \theta} = \frac{1}{n} \sum_{i=1}^n \frac{\partial L(\theta)}{\partial \theta} = 0$$

The above equation is linear in the parameters of the linear model.

At this point, it is clear that

$$L(\theta) = \frac{1}{n} \sum_{i=1}^n L(\theta_i)$$

Then, by using the fact that the function is strictly convex

the above equation is linear in the parameters of the linear model.

$$(5-2) \quad \frac{\partial L(\theta)}{\partial \theta} = \frac{1}{n} \sum_{i=1}^n \frac{\partial L(\theta_i)}{\partial \theta} = 0$$

The above equation is linear in the parameters of the linear model.

At this point, it is clear that

CHAPTER V

CONCLUSIONS

The problem of phase measurement and control is of great importance to the armed forces and industry alike. A great deal of work has been done and much is now going on, a large portion of which must be assumed to be guarded by classification. Ultimately the quality of phase may assume the same importance in the electronic field as is now held by frequency. To this end and solely on the basis of the general unclassified matter presented in this paper the following conclusions are drawn.

1) There exists a need for a broad and phase shift network which is not as critically dependent upon component values as those presently available.

2) The range of frequencies of the existing phase shift networks should be extended.

3) A precision phase shifter which is frequency insensitive would be the ideal method to employ in the measurement of phase. In this way every value of shift could be obtained by the null method which is inherently the most accurate means of electrical measurement available.

4) A theoretical study as to whether phase discrimination is most accurately and economically done at microwave frequencies with magic T's and rat-races or at some intermediate frequency after heterodyning might be useful.

5) A reliable commercial phase meter with an accuracy of plus or minus 1 degree and a range of 0.1 to 1000 kilocycles obtained with a reasonable number of controls should prove a marketable item.

6) It seems unlikely any new circuits or techniques for the measurement or control of phase will be forthcoming. Any improvement must be by refinement of existing circuitry or different application of existing techniques.

7) As a precision broadband phase measuring device the instrument outlined in Figure 29 is suggested. Quadrature voltages for a precision goniometer of either the resistance or capacitance type are obtained from broadband phase shift networks. The goniometer is used to insert a precision phase shift in series with one of the signals to be compared to bring about phase coincidence. The phase null indicator is a diode bridge circuit since this circuit will suitably as a phase null indicator with complex waveforms. Amplitude variations are removed by a series of clipper circuits.

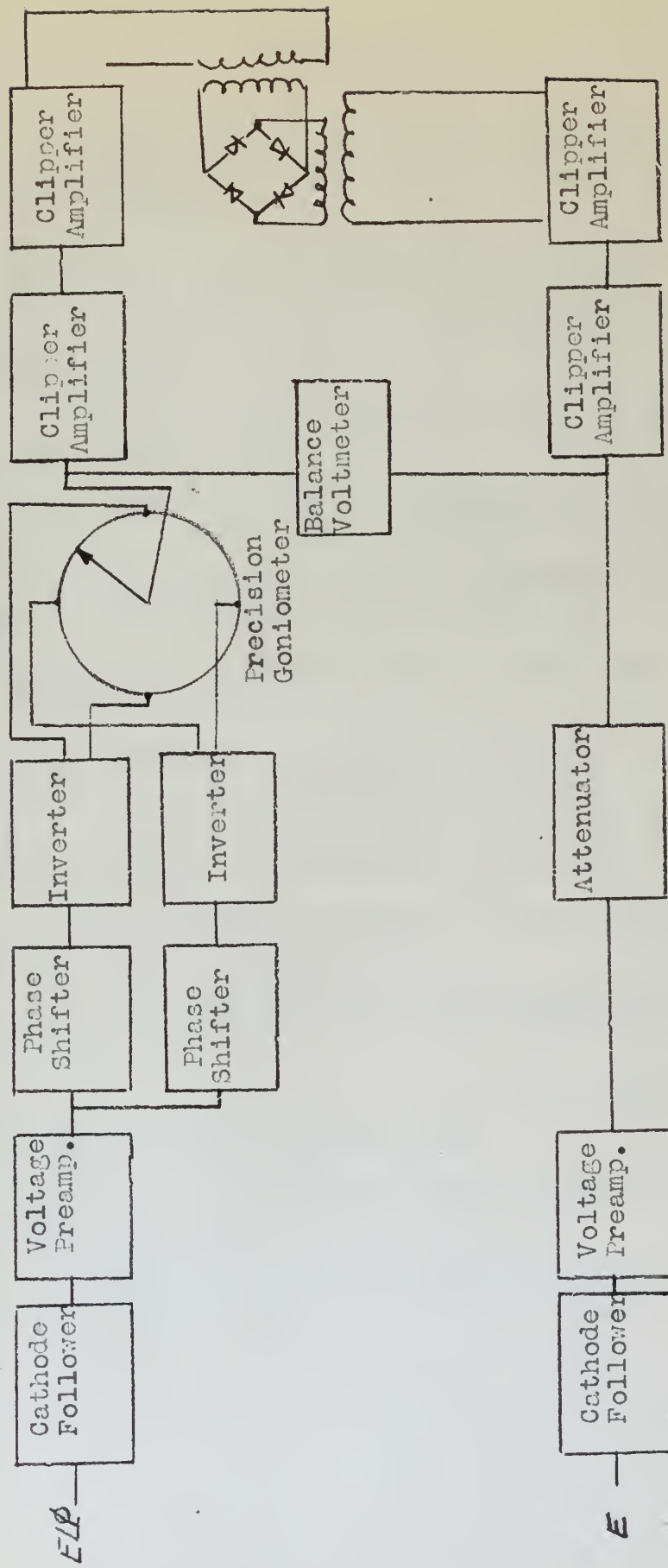


Figure 29

Proposed Precision Phasemeter

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APPENDIX I

A Precision Measurement of Phase Coincidence Using a Cathode Ray Oscilloscope

This method of obtaining precise measurements around phase coincidence is abstracted^c from a method of production testing in use by a large electronics concern. Because of the numerous incidences encountered in collecting the bibliographical material for this thesis where this method has been used as a laboratory procedure it is considered sufficiently important to be included as an appendix.

Given the problem of measuring the exact condition of phase coincidence or slight errors therefrom:

1. A CRO is reworked to provide a high degree of linearity and similarity in its deflection amplifiers as evidenced by standard tests performed on these amplifiers.
2. The two signals to be compared in phase are applied to the vertical and horizontal deflection amplifiers respectively producing the straight line pattern characteristic of this condition.
3. The signal levels are recorded and each increased by a like factor. This puts the pattern off the scope face, but considering the deflection to be still linear the maximum trace excursion can still be considered as being the original excursion multiplied by the voltage factor.
4. Small vertical and horizontal intercepts indicating departures from phase coincidence previously not observable can now be measured.

A further statement of the author is given in the following

conclusion

This is a very interesting and important question, and it is

very difficult to answer. It is a question of the

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APPENDIX II

Phase Invariance During the Heterodyning Process

For the general case of plate current as a function of grid voltage

$$i_p = g_m \left(e_g + \frac{e_b}{\mu} \right) + \frac{1}{2} \frac{\partial g_m}{\partial e_g} \left(e_g + \frac{e_b}{\mu} \right)^2 + \frac{\partial^2 g_m}{\partial e_g^2} \frac{1}{3!} \left(e_g + \frac{e_b}{\mu} \right)^3 + \text{etc}$$

If C_b can be described as having a constant relationship to which amounts to saying that μ remains constant the expression becomes

$$i_p = g_m k e_g + \frac{\partial g_m}{\partial e_g} \frac{k^2}{2!} e_g^2 + \frac{\partial^2 g_m}{\partial e_g^2} \frac{k^3}{3!} e_g^3 + \dots \text{etc.}$$

and the variation of with is normally sufficiently small to make the larger power terms vanishingly small. This expression shows that there is a value of plate current which is proportional to the square of the grid voltage. Square law conversion, as it is called, is more efficiently accomplished by choosing proper circuit parameters to produce a parabolic transfer characteristic from which the expression for plate current become directly

$$i_p = a e_g^2$$

By applying the usual trigonometric reduction formulas to

$$i_p = a [A \cos(\omega_c t + \phi) + B \cos(\omega_o t)] \quad \omega_c > \omega_o$$

an expression for the different frequency components is obtained

$$i_p = \frac{a A^2}{2} + \frac{a A^2}{2} \cos 2(\omega_c t + \phi) + 2 \frac{a A B}{2} \cos(\omega_c t + \phi + \omega_o t) \\ + 2 \frac{a A B}{2} (\omega_c t + \phi - \omega_o t) + \frac{a B^2}{2} \cos(2 \omega_o t) + \frac{a B^2}{2}$$

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$$f(x) = \frac{1}{2} \left(\frac{d^2}{dx^2} + \frac{1}{x} \frac{d}{dx} \right) \left(\frac{d^2}{dx^2} + \frac{1}{x} \frac{d}{dx} \right) f(x)$$

the following is a list of the

$$f(x) = \frac{1}{2} \left(\frac{d^2}{dx^2} + \frac{1}{x} \frac{d}{dx} \right) \left(\frac{d^2}{dx^2} + \frac{1}{x} \frac{d}{dx} \right) f(x)$$

If sufficiently selective circuits are postulated to remove the unwanted frequency components, the desired intermediate frequency is

$$aAB \cos[(\omega_c - \omega_0)t + \phi]$$

In the general conversion case where the two signals may be coupled in the electron beam it is customary to postulate that the local oscillator amplitude is very much greater than the signal amplitude and that the tube transconductance becomes a periodically varying function of the local oscillator signal amplitude. By expressing transconductance as a Fourier Series

$$g_m = \frac{b_0}{2} + b_1 \cos \omega_0 t + b_2 \cos 2\omega_0 t + \dots$$

and the a-c plate current due to the signal voltage is

$$i_p = g_m E_{sig} \cos(\omega_c t + \phi)$$

Again removing all unwanted higher and lower frequency terms by use of selective circuits the desired intermediate frequency becomes

$$\begin{aligned} i_p &= b_1 E_{sig} \cos(\omega_c t + \phi) \cos \omega_0 t \\ &= \frac{b_1 E_{sig}}{2} \cos[(\omega_c - \omega_0)t + \phi] + \dots \end{aligned}$$

APPENDIX III

Analysis of the Cathode Coupled Clipper Circuit

This circuit is illustrated in Figure 16. Complete development of this circuit was originally done as a government sponsored research project and is completely discussed in reference 6 . Only the portions of that article describing the action of the basic circuit as a clipper are treated here. The concept of negative resistance is analogous to positive feedback and, as might be suspected when the concept is evolved out of gain considerations, positive feedback can affect the operation as a clipper. The effects of this type of feedback are treated in the original article.

It is often the practice in the use of this circuit to cascade several clipping stages, perhaps with interspersed amplification stages, when the available signal is very low and very steep wave fronts are desired. In this case any additional gain available in the clipper stages can be used. To this end the balance of this appendix is devoted.

The circuit can be thought of as a cathode follower, the output of which drives a plate load triode amplifier by grid injection. The first stage is cut off by negative grid signal. Since the grid of the triode amplifier is at a.c. ground and fixed d.c. potential a positive going grid signal soon raises the cathode voltage and the second tube is cut off. For negative going grid signal the second tube hastens the time of cut off by tending to keep the common cathode at a high potential.

If the upper and lower voltage excursions are designated $e (+)$ and $e (-)$ the following relationships can be developed for the clipped wave, Figure 16.

$$\text{Time A-B} = T = 1/f = 2\pi/\omega$$

$$e(t) = E_{sm} \sin \omega t \doteq E_{sm} \omega t \quad t_1 \leq t \leq t_2$$

Therefore in the time interval a-b the instantaneous voltage is, since the sine of the angle is practically equal to the angle

$$e(t) = E_{sm} \omega t + E_{cc1} \quad ; \quad e(-) = -E_{sm} \omega t + E_{cc1}$$

$$e(t) - e(-) = E_{sm} (t_2 - t_1) \omega = E_{sm} t \omega$$

$$\% \text{ Rise Time} = t/T \times 100 \quad ; \quad t = \frac{e(t) - e(-)}{E_{sm} \omega}$$

$$= \frac{e(t) - e(-)}{E_{sm} \omega} \times 100 = \frac{e(t) - e(-)}{2\pi E_{sm}} \times 100$$

The smaller the rise time the less the possibilities dissymmetry. If a tube parameter μ' which is defined as the ratio of plate voltage to grid voltage at plate cut off is introduced the values of $e (+)$ and $e (-)$ become

$$e(t) \approx E_{cc2} + E_{bb} \frac{(\mu - \mu')}{\mu \mu'}$$

$$e(-) \approx E_{cc2} - E_{bb} \frac{(\mu - \mu')}{\mu \mu'}$$

The figure of merit of this circuit is the "input ratio". This is the ratio of the maximum grid to ground signal which may be applied without causing grid current to flow to the minimum grid to ground signal at which clipping just begins.

It is found that the voltage across the resistor is 10 V and the voltage across the capacitor is 20 V. The voltage across the inductor is 30 V. The voltage across the source is 40 V.

$$V_s = 40 \text{ V}$$

$$V_R = 10 \text{ V}, V_C = 20 \text{ V}, V_L = 30 \text{ V}$$

The voltage across the resistor is 10 V, the voltage across the capacitor is 20 V, and the voltage across the inductor is 30 V. The voltage across the source is 40 V.

$$V_R = 10 \text{ V}, V_C = 20 \text{ V}, V_L = 30 \text{ V}$$

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$$\begin{aligned} \text{Input Ratio (I.R.)} &= \frac{e_{\max} - E_{cc1}}{e(t) - E_{cc1}} = \frac{E_{sm}}{e(t) - E_{cc1}} \\ &= \frac{\mu\mu'}{E_{bb}(\mu - \mu')} \left[\frac{E_{bb}R_k}{R_k + r_p} + \frac{E_{bb}}{\mu} - \frac{E_o R_k}{R_e} \right] \end{aligned}$$

[E_o is output amplitude (peak)]

The characteristics of the next stage determine the amplitude of the permissible signal which may be applied, that is the clipper output pulse. The total input-output capacity and the frequency requirements determine the size of the plate resistor in the manner of any other RC amplifier. With R_e and E_o fixed the expression for Input Ratio is maximized to give the following values for R_k and E_{cc1} 2. The two grid biases are chosen to be equal to prevent phase dissymmetry distortion discussed in the text.

$$\begin{aligned} R_k &= \sqrt{\frac{E_{bb} R_e r_p}{E_o}} - r_p \\ E_{cc1} = E_{cc2} &= \sqrt{\frac{E_{bb} E_o r_p}{R_e}} - \frac{E_{bb}}{\mu} - \frac{E_o r_p}{R_e} \end{aligned}$$

The relationship of the input ration to the rise time is

$$\text{Rise time (\%)} = \frac{e(t) - e(-)}{2\pi E_{sm}} \times 100$$

$$\text{Since } e(-) = 2 E_{cc} - e(t)$$

$$\text{Rise time} = 100 \left\{ \frac{2e(t) - 2E_{cc}}{2\pi E_{sm}} \right\} = \frac{100}{\pi \left[\frac{E_{sm}}{e(t) - E_{cc}} \right]}$$

$$= 100 / \pi (\text{I.R.})$$

Which indicates that for optimum clipper action with minimum rise time maximum input ratio should be used.

The problem is now to reconcile performance as a clipper with best

performance as a linear amplifier. The Taylor series expansion for the incremental plate current is

$$i_p = g_m(e_g + \frac{e_b}{\mu}) + \frac{1}{2} \frac{\partial g_m}{\partial e_g} (e_g + \frac{e_b}{\mu})^2 + \frac{1}{6} \frac{\partial^2 g_m}{\partial e_g^2} (e_g + \frac{e_b}{\mu})^3 + \dots$$

and to a first approximation

$$i_p = g_m e_g + \frac{e_b}{r_p}$$

The following relationships can be developed.

1. $e_{c1} = E \sin \omega t - e_k$
2. $e_k = (i_{p1} + i_{p2}) R_k$
3. $e_{c2} = -e_k$
4. $i_{p1} = g_m e_{c1} + \frac{e_{b1}}{r_p} = g_m e_{c1} - \frac{(i_{p1} + i_{p2}) R_k}{r_p}$
5. $i_{p2} = g_m e_{c2} + \frac{e_{b2}}{r_p} = g_m e_{c2} - \frac{(i_{p1} + i_{p2}) R_k}{r_p} - \frac{i_{p2} R_e}{r_p}$
6. $e_{out} = i_{p2} R_e$

Solving for the plate currents and output voltage

$$\begin{aligned} \text{I} \quad i_{p1} &= g_m E \sin \omega t - g_m R_k (i_{p1} + i_{p2}) - \frac{(i_{p1} + i_{p2}) R_k}{r_p} \\ \text{II} \quad i_{p2} &= -g_m R_k (i_{p1} + i_{p2}) - \frac{(i_{p1} + i_{p2}) R_k}{r_p} - \frac{i_{p2} R_e}{r_p} \end{aligned}$$

Eliminating i_{p1} and i_{p2} between these equations the expression for gain is obtained as a function only of the circuit parameters.

$$(1 + g_m R_k + R_k/r_p) i_{p1} + (g_m R_k + R_k/r_p) i_{p2} = g_m E \sin \omega t$$

$$(g_m R_k + R_k/r_p) i_{p1} + (1 + g_m R_k + R_e/r_p) i_{p2} = 0$$

$$i_{p2} = - \frac{[g_m E \sin \omega t (g_m R_k + R_k/r_p)]}{[1 + g_m R_k + R_k/r_p][1 + g_m R_k + R_e/r_p] - [g_m R_k + R_k/r_p]^2}$$

$$\frac{e_{out}}{E(\sin \omega t)} = - \frac{[g_m^2 R_e R_k + g_m R_k R_e/r_p] r_p^2}{[r_p^2 + (2\mu + 1) R_k r_p + (\mu + 1) R_e R_k - (\mu + 1) R_k^2 + r_p R_e]}$$

$$\tilde{K} = - \frac{(\mu+1)\mu R_k R_e}{\mu \left(\frac{r_p}{g_m} + \frac{R_e}{g_m} \right) + (2\mu+1) R_k r_p + (\mu+1) R_e R_k - (\mu+1) R_k^2}$$

$$\mu+1 \doteq \mu \quad ; \quad 2\mu+1 \doteq 2\mu$$

$$\tilde{K} = - \frac{\mu R_k R_e}{\left(\frac{r_p}{g_m} + \frac{R_e}{g_m} \right) + 2 R_k r_p + R_e R_k - R_k^2}$$

If an attempt is made to maximize this gain expression for R_k

$$\frac{\partial K}{\partial R_k} = \frac{\mu R_e \left\{ \left[\frac{r_p + R_e}{g_m} + 2 R_k r_p + R_e R_k - R_k^2 \right] - R_k [2 r_p + R_e - 2 R_k] \right\}}{\left[\dots \dots \dots \right]^2} = 0$$

$$\frac{r_p + R_e}{g_m} + R_k^2 = 0$$

$$R_k = \pm j \sqrt{\frac{r_p + R_e}{g_m}}$$

The expression could also be made to hold for a negative value for r_p .

$$\frac{25 \times 10^{-3} \times (1 + 10^{-3})}{25 \times 10^{-3} - 10^{-3} \times 10^{-3}} = 1.0001$$

$$10^{-3} \times 10^{-3} = 10^{-6}$$

$$\frac{25 \times 10^{-3} \times 10^{-3}}{25 \times 10^{-3} - 10^{-6}} = 1.0001$$

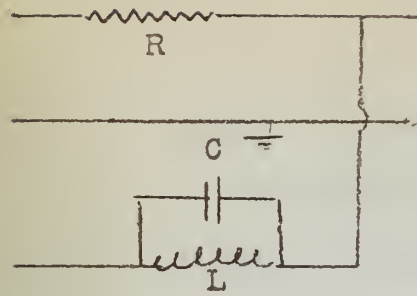
It is evident that the value of α is very small.

$$\frac{[25 \times 10^{-3} - 10^{-3} \times 10^{-3}] \times 10^{-3}}{[25 \times 10^{-3} - 10^{-6}]} = \frac{25 \times 10^{-6}}{25 \times 10^{-3} - 10^{-6}}$$

$$\frac{25 \times 10^{-6}}{25 \times 10^{-3} - 10^{-6}} = 10^{-3}$$

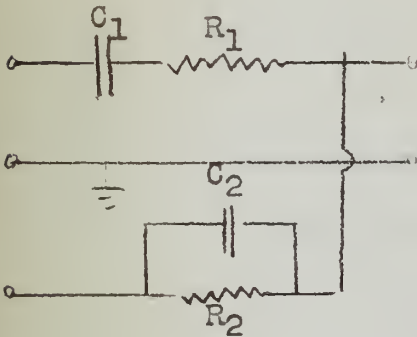
The expression could also be used for a negative value of α .

Reduction of Several Phase Shift Networks to Basic Form

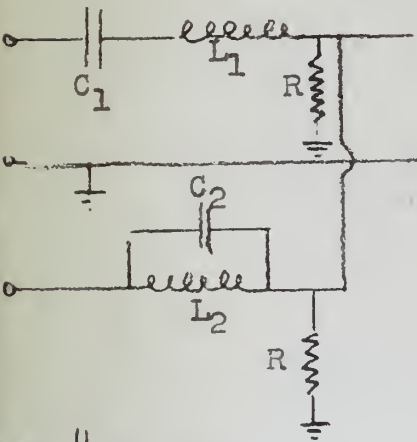


$$\begin{aligned} \omega^2 &= 1/LC \\ Q &= CR/\omega L \\ k &= 1/2 \\ L &= R/\omega Q \\ C &= 2/\omega R \end{aligned}$$

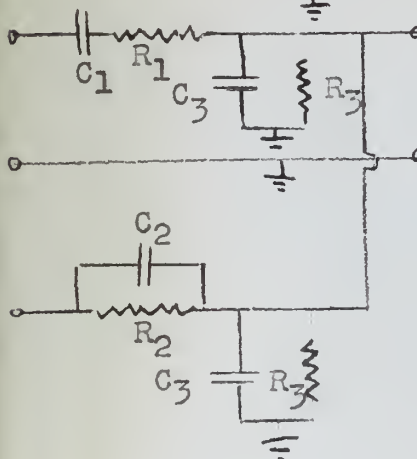
ω is resonant frequency and subscript omitted in notation



$$\begin{aligned} \omega &= 1/R, C_1 = 1/R_2 C_2 \\ Q &= 1/R_2 R_1 + 2 \\ k &= 1 - 2Q \\ R_1 &= R_2 / (1/Q - 2) \\ C_2 &= 1/\omega R \\ C_1 &= (1/Q - 2)/\omega R \end{aligned}$$



$$\begin{aligned} \omega^2 &= 1/L_1 C_1 = 1/L_2 C_2 \\ Q &= L_1/C_1 R^2 = C_2 R/L_2 \\ k &= 1/2 \\ L_1 &= QR/\omega \\ C_1 &= R/\omega R \\ L_2 &= R/\omega Q \\ C_2 &= Q/\omega R \end{aligned}$$



$$\begin{aligned} \omega &= 1/R, C_1 = 1/R_2 C_2 = 1/R_3 C_3 \\ Q &= 1/(R_2/R_1 - 2) = 1/R_3/R_2 + 2 \\ k &= (1 - 2Q)/(1 + 2Q) \\ R_2 &= R_3/(1/Q - 2) \\ R_1 &= R_3/(1/Q - 4) \\ C_3 &= 1/\omega R \\ C_2 &= (1/Q - 2)/\omega R_2 \\ C_1 &= (1/Q - 4)/\omega R_3 \end{aligned}$$

APPENDIX V

Precision Capacitance Goniometer

The physical configuration of this device and its equivalent circuit are shown in Figure 28.

Writing the expression for the output current

$$i_{out} = \frac{e}{Z} = \frac{e_1 - e}{1/j\omega C_1} + \frac{e_2 - e}{1/j\omega C_2} + \frac{e_3 - e}{1/j\omega C_3} + \frac{e_4 - e}{1/j\omega C_4}$$

The expression for the branch currents reduce to

$$e/j\omega Z = e_1 C_1 + e_2 C_2 + e_3 C_3 + e_4 C_4 - e(C_1 + C_2 + C_3 + C_4)$$

$$\begin{aligned} e_1 C_1 &= E_m \sin \omega t (C_0 + C_m \cos \theta) \\ &= E_m C_0 \sin \omega t + \frac{E_m C_m}{2} \{ \sin(\omega t + \theta) + \cos(\omega t - \theta) \} \end{aligned}$$

$$e_2 C_2 = -E_m C_0 \cos \omega t + \frac{E_m C_m}{2} \{ \sin(\omega t + \theta) - \cos(\omega t - \theta) \}$$

$$e_3 C_3 = -E_m C_0 \sin \omega t + \frac{E_m C_m}{2} \{ \sin(\omega t + \theta) + \sin(\omega t - \theta) \}$$

$$e_4 C_4 = E_m C_0 \cos \omega t + \frac{E_m C_m}{2} \{ \sin(\omega t + \theta) - \sin(\omega t - \theta) \}$$

$$e/j\omega Z = 2 E_m C_m \sin(\omega t + \theta) - 4 e C_0$$

Therefore the voltage across the load becomes

$$e = \frac{j\omega Z}{4C_0 + j\omega Z} \{ 2 E_m C_m \sin(\omega t + \theta) \}$$

PROBLEM 7

Consider the following circuit:

The input voltage is $v_i(t) = 10 \cos(1000t)$ V and the output voltage is $v_o(t)$.

Find the average power in the output voltage.

Hint: Use the average power formula.

$$v_i(t) = 10 \cos(1000t) \text{ V}$$

The average power in the output voltage is

$$P_{avg} = \frac{1}{T} \int_0^T v_o(t) i_o(t) dt$$

$$= \frac{1}{T} \int_0^T v_o(t) i_o(t) dt$$

$$= \frac{1}{T} \int_0^T v_o(t) i_o(t) dt$$

$$= \frac{1}{T} \int_0^T v_o(t) i_o(t) dt$$

$$= \frac{1}{T} \int_0^T v_o(t) i_o(t) dt$$

$$= \frac{1}{T} \int_0^T v_o(t) i_o(t) dt$$

$$= \frac{1}{T} \int_0^T v_o(t) i_o(t) dt$$

Therefore the average power in the output voltage is

$$P_{avg} = \frac{1}{T} \int_0^T v_o(t) i_o(t) dt$$

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